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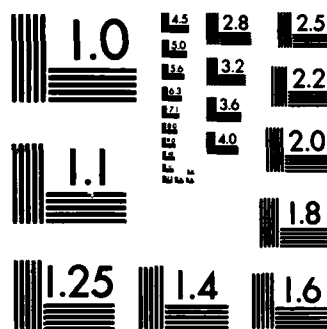
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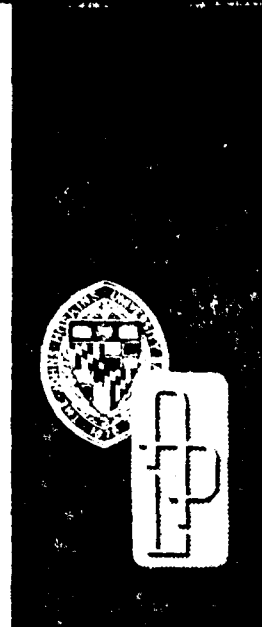
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*Technical Memorandum*

# ISSUES CRITICAL TO THE APPLICATION OF ADAPTIVE ARRAY ANTENNAS TO MISSILE SEEKERS

R. L. TRAPP

C. H. RONNENBURG

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## ABSTRACT

Missile seekers will confront complex and hostile signal environments that can inhibit severely their ability to intercept threatening targets. Dramatic target detection and homing performance improvement in main beam and sidelobe jamming is realizable with a seeker antenna that can optimally adapt, in real time, its response to the signal environment. Adaptive array antennas can be designed to optimize the signal-to-interference-plus-noise ratio by forming pattern nulls directed toward sources of interference while simultaneously maximizing gain in the desired signal direction.

Physical and operational missile constraints place severe requirements on an adaptive array. Nevertheless, there are several array configurations and adaptive processors that can satisfy these constraints in the next decade. Technology is a dominant limitation to adaptive array performance in a missile seeker. Signal processors and array implementations using state-of-the-art technology are required. Critical experimentation and representative simulations are needed to establish error effects, preferred adaptive array implementations, detailed requirements, and relative cost estimates. Although an adaptive missile seeker antenna is physically realizable in the next decade, the tradeoffs between cost, complexity, and performance will determine its utility and practicality.

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## 1. SUMMARY

Missile seekers with fixed pattern antennas are unduly susceptible to main beam and sidelobe jamming from standoff jammers. Although sidelobe levels can be reduced to overcome sidelobe jamming, the concomitant broadening of the main beam then increases the likelihood of main beam jamming. Reducing the beamwidth of the main beam by raising the missile guidance frequency can reduce that probability. However, a higher guidance frequency can result in significantly degraded all-weather performance and increased target search times. No solution is, in itself, completely satisfactory.

Significantly improved suppression of main beam and sidelobe jamming can be achieved by adapting (in real time) the pattern characteristics of a missile seeker array antenna in response to the signal environment. Adaptive array antennas optimize signal-to-interference-plus-noise ratio (S/IN) by sensing the signal environment, then forming pattern nulls in jammer directions and maximizing gain in the desired signal direction. Target angle estimation can also be optimized when an adaptive array antenna is configured with multiple beams. Although improved performance against jamming is gained without an increase in guidance frequency, the complexity and cost of an adaptive array antenna is considerably greater than that of a more conventional fixed pattern antenna. An adaptive array antenna requires a processor to compute the optimum weights for an array of sensor elements and a variable beam-forming network to achieve the proper weighting and combining of sensor elements into one or more beams.

S/IN improvements of well over 10 dB relative to fixed pattern antennas are predicted by adaptive array theory in a number of jamming environments. When searching for a randomly located target against a background of jammers that completely blinds a conventional array, an adaptive array is shown to have a significant probability of target detection. An adaptive array is capable of continued target tracking when main beam jamming occurs. Conventional arrays are susceptible to "capture" by a main beam jammer, resulting in jammer tracking instead of desired target tracking. Thus, an adaptive missile seeker antenna offers the potential for greatly improved target detection and homing performance in severe interference environments. Possible limiting

factors on adaptive array performance include signal bandwidth, multipath propagation, "smart" jammer techniques, antenna array errors, adaptive processor limitations, and beam-forming network errors.

The general requirements for an adaptive missile seeker antenna are established by assuming a high performance air intercept missile application. In addition to the missile's physical constraints, the adaptive missile seeker antenna is limited to the signal processing capability forecast for the 1990's. Hypersonic closing velocities and rapid target search requirements severely limit the maximum time available to adaptively form a sum beam and two difference beams.

Several antenna array configurations have been evaluated for adaptive missile seeker antenna suitability. Although a fully adaptive planar array is desirable from a performance perspective, the complexity of its implementation and processing requirements are excessive for the assumed 1990's missile seeker. However, partially adaptive array techniques show promise for practical missile implementation. Adaptively controlling antenna elements in groups or subarrays not only reduce feed network complexity, but also significantly reduces the signal processing requirement. Partially adaptive arrays are generally more susceptible to errors than fully adaptive arrays and are not as capable in extremely dense jamming environments. Even so, the performance of a partially adaptive array approaches that of a fully adaptive array under many conditions.

The heart of the adaptive processor is its control algorithm. The processor attributes, including its complexity and transient response speed, are established primarily by the adaptive algorithm. A number of adaptive algorithms are evaluated for practical adaptive processor implementation in a missile seeker relative to the established requirements. The digital adaptive processors that appear to be best suited for missile seekers are those based on algorithms that take the inverse of a sample covariance matrix of sensor outputs. Although these algorithms are computation intensive and will require a state-of-the-art digital processor, the response speed is adequate for a missile seeker.

Adaptive missile seeker antennas appear to be realizable in the next decade. The potential for improved target detection and homing performance is consider-

able. However, further investigation, development, and experimentation are needed to accurately estab-

lish the tradeoffs between cost, complexity, and performance.

## 2. INTRODUCTION

An adaptive array antenna senses the signal environment and automatically adjusts the array characteristics to optimize its performance. Typically, an adaptive antenna system consists of an array of variably weighted antenna elements and an adaptive processor. According to a selected algorithm, the adaptive processor estimates, in real time, the signal environment from the element outputs and computes a set of optimum element weights that will form a desired antenna response. The S/IN is optimized when an adaptive antenna places pattern nulls in directions corresponding to interference while simultaneously maximizing gain in the direction of the desired signal. In addition to optimally filtering in the spatial domain, adaptive array techniques can be used to optimize an antenna's frequency and polarization response.

Most of the adaptive array theory has been developed since 1960.<sup>1</sup> Fundamental contributions by

Howells, Applebaum, and Widrow<sup>2,3</sup> were the catalysts to this now intensely active area. Currently, adaptive arrays are being applied to a wide variety of communication, radar, and sonar systems.<sup>4</sup> Adaptive communication antennas are being developed for satellites, aircraft, data links, and more.<sup>7-11</sup> Radar applications include sidelobe cancellers in search radars, low-angle tracking, airborne MTI radars, and satellite-based radars.<sup>10-16</sup> Yet there has been little effort to apply an adaptive array to a missile seeker.

The applicability of an adaptive array antenna to a missile seeker should be addressed, since it could yield significantly improved target acquisition and homing performance. Can a practical and cost-effective implementation of an adaptive missile seeker antenna be developed that achieves some desired level of performance improvement over a more conventional seeker antenna? This report identifies and describes the issues critical to that question.

<sup>1</sup>W. F. Gabriel, "Adaptive Arrays—An Introduction," *Proc. IEEE*, **64**, pp. 239-272 (Feb 1976).

<sup>2</sup>P. W. Howells, "Intermediate Frequency Side-Lobe Canceller," U.S. Patent 3202 990 (24 Aug 1965) (filed May 4, 1959).

<sup>3</sup>P. W. Howells, "Explorations in Fixed and Adaptive Resolution at GE and SURC," *IEEE Trans. Ant. Propag.*, **AP-24**, pp. 575-584 (Sep 1976).

<sup>4</sup>S. P. Applebaum, "Adaptive Arrays," *IEEE Trans. Ant. Propag.*, **AP-24**, pp. 585-598 (Sep 1976).

<sup>5</sup>B. Widrow, P. E. Mantz, L. J. Griffiths, and B. B. Goode, "Adaptive Antenna Systems," *Proc. IEEE*, **55**, pp. 2143-2159 (Dec 1967).

<sup>6</sup>P. M. Grant and C. F. N. Cowen, "Adaptive Antennas Find Military and Civilian Applications," *Microwave Sys. News*, pp. 97-107 (Sep 1981).

<sup>7</sup>R. T. Compton, Jr., R. J. Huff, W. G. Swarner, and A. A. Ksiowski, "Adaptive Arrays for Communication Systems: An Overview of Research at the Ohio State University," *IEEE Trans. Ant. Propag.*, **AP-24**, pp. 599-607 (Sep 1976).

<sup>8</sup>R. T. Compton, Jr., "An Adaptive Array in a Spread-Spectrum Communication System," *Proc. IEEE*, **66**, pp. 289-298 (Mar 1978).

<sup>9</sup>L. E. Brennan and I. S. Reed, "An Adaptive Array Signal Processing Algorithm for Communications," *IEEE Trans. Aerosp. Electron. Sys.*, **AES-18**, pp. 124-130 (Jan 1982).

<sup>10</sup>*Proc. 1980 Adaptive Ant. Symp.*, Rome Air Development Center, RADC-TR-378, Vol. 1-III (Dec 1980).

<sup>11</sup>L. E. Brennan and I. S. Reed, "Theory of Adaptive Radar," *IEEE Trans. Aerosp. Electron. Sys.*, **AES-9**, pp. 237-252 (Mar 1973).

<sup>12</sup>L. E. Brennan, J. D. Mallett, and I. S. Reed, "Adaptive Arrays in Airborne MTI Radar," *IEEE Trans. Ant. Propag.*, **AP-24** (Sep 1976).

<sup>13</sup>P. J. Baldwin, E. Denison, and S. F. O'Connor, "An Experimental Analogue Adaptive Array for Radar Applications," *IEEE 1980 Int. Radar Conf. Record* (Apr 1980).

<sup>14</sup>E. C. DuFort, "An Adaptive Low-Angle Tracking System," *IEEE Trans. Ant. Propag.*, **AP-29**, pp. 766-772 (Sep 1981).

<sup>15</sup>L. J. Griffiths, "Time-Domain Adaptive Beamforming of HF Backscatter Radar Signals," *IEEE Trans. Ant. Propag.*, **AP-24**, pp. 707-720 (Sep 1976).

<sup>16</sup>J. W. McIntyre, R. E. Down, and V. I. von Mehlem, "Satellite Surveillance System with Closed Loop Array Beam Forming (Task X)," JHU: APL S3C-1-105 (24 Aug 1981).

### 3. MOTIVATION

To intercept a target, a missile must acquire the target with its radar (active) or receiver (semiactive), initiate tracking, and guide itself to the target. Target acquisition generally requires a search over a specified spatial volume (elevation angle, azimuth angle, and range) until an appropriate target can be detected. Once a target is detected, the target angle relative to the seeker's antenna boresight is estimated and tracking is initiated. The missile is then directed to an intercept point in accordance with some guidance algorithm (e.g., proportional navigation) and missile control technique.

The seeker's antenna is vital to each phase of missile seeker operation. During acquisition the spatial search volume must be searched rapidly while maintaining a maximum probability of target detection. Jamming or other interference can degrade the target detection probability. The susceptibility of a missile seeker to interference is determined to a great extent by its antenna pattern. The angle estimation that occurs during tracking typically requires multiple antenna beams. In a monopulse seeker, a sum beam and two difference beams (elevation and azimuth) are common. The proper formation of these beams and minimal interference are essential to the accurate estimation of target position. When the missile is proceeding toward intercept, the target continues to be tracked. The missile's flight is controlled as a function of estimated target position.

Missile seekers are required to operate effectively in severe environments, yet with a variety of limitations. The relatively small size of a missile seeker limits the antenna aperture and, correspondingly, the antenna beamwidth for a given operating frequency. Signal processing capability is also limited by allowable weight and power. The constantly changing signal environment typically includes a desired signal, thermal noise (generated in the receiver), clutter returns, and interference. The signal environment is complicated further with radome distortion and multipath propagation. Usually the desired signal level is low and competes with other signals that are much larger. Clutter is generally separated from the desired signal by Doppler filtering. However, seeker platform motion and other conditions make this separation more difficult. Interference can be unintentional or deliberate.

An adaptive antenna can be used to compensate for undesired signals. Most of the current interest in

adaptive antennas is a result of their ability to suppress interference. It is this interference suppression capability that makes adaptive antennas a potential source of significant performance improvement in a missile seeker.

One can postulate a number of jamming threats to a missile seeker. A particularly disturbing threat consists of multiple standoff aircraft jamming at the guidance frequency and distributed over a given sector so as to mask incoming missiles and launch aircraft. Positioned at or beyond the missile's range, standoff jammers can inhibit severely the missile's ability to find and destroy threatening targets. A very low sidelobe seeker antenna can help alleviate this problem, but it is susceptible to main beam jamming. In fact, reducing the sidelobes of an antenna increases the width of the main beam, which in turn increases the likelihood of main beam jamming. Figure 1 illustrates this effect by plotting Dolph-Chebyshev and uniform array patterns for an 8-element  $\lambda/2$ -spaced linear array. The 30 dB Dolph-Chebyshev pattern is the optimum in-phase distribution in the sense that the beamwidth is minimum for a given sidelobe level. Although the 3 dB beamwidths between these two arrays differ only slightly, it is the null-to-null beamwidths that more accurately reflect their main beam jamming susceptibility. The Dolph-Chebyshev array has a null-to-null beamwidth 50% greater than that of the uniform array. The patterns in Fig. 1 correspond to the sum pattern of a monopulse seeker antenna. The difference patterns can be even more susceptible to jamming in the main difference pattern lobes, since they typically extend beyond the sum pattern main beam. In addition, the sidelobes of the difference patterns are generally higher than the sum pattern sidelobes unless an independent aperture distribution such as that described by Bayliss<sup>17</sup> is used.

Another method of improving interference suppression is to use a narrower beam antenna in addition to designing for low sidelobes. Main beam beamwidth is inversely proportional to frequency for a constant antenna aperture. Since a missile seeker is generally antenna aperture limited, the beamwidth of the main beam can be reduced only by increasing the

<sup>17</sup>E. T. Bayliss, "Design of Monopulse Antenna Difference Patterns with Low Sidelobes," *The Bell Sys. Tech. J.*, pp. 623-650 (May-Jun 1968).

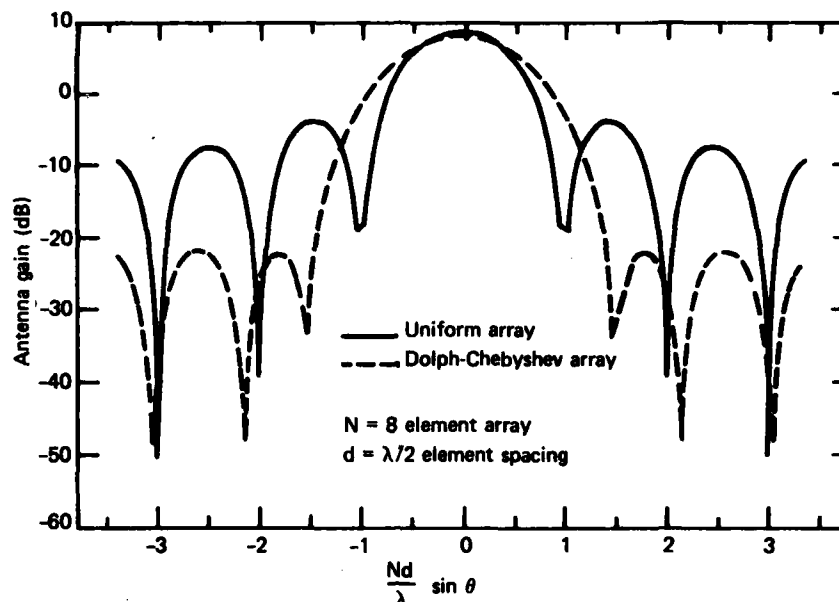


Figure 1 — Dolph-Chebyshev and uniform linear array antenna patterns.

operating frequency. While this is an effective method for reducing the probability of main beam or near-in sidelobe jamming, it can adversely affect performance. Atmospheric attenuation, rain attenuation, sea backscatter, and rain backscatter increase dramatically with frequency and can degrade significantly the all-weather performance of a missile seeker. Available transmitter power and efficiency often decreases with increasing frequency. In addition, a narrower beam antenna increases the time required to search for a target in a given spatial volume.

The optimal solution to main beam and sidelobe jamming is an antenna system that adjusts itself in spatial, spectral, and polarization response to the signal environment as it changes. In addition to adjusting the monopulse sum beam for optimal target detection during target acquisition, the sum and difference beams are adjusted for optimal target angle estimation when tracking is initiated. An adaptive antenna is theoretically capable of such optimization.

An adaptive array sum beam that maximizes the S/I/N places antenna pattern nulls in jammer directions and maximizes gain in the target direction.<sup>4</sup> In addition to all but eliminating sidelobe jammer degradation, main beam jamming is effectively suppressed, reducing the potential for successful main beam jamming.

An adaptive missile seeker antenna is considerably more complex than a conventional seeker antenna. The adaptive antenna requires an array with a con-

trollable beam-forming network for each independently formed beam, and an adaptive processor. A generic adaptive array antenna with single beam formation is illustrated in Fig. 2. The adaptive processor is essentially a multichannel receiver that implements an adaptive control algorithm. It typically estimates the signal environment by correlating the individual array element outputs in real time. The beam-forming weights are computed from the signal environment estimate according to the selected algorithm. Since the adaptive processor must sense the environment and then react, there is an associated transient response time between a change in signal environment and realization of the optimum antenna response. An adaptive array is generally configured to optimize signal reception in a given pointing direction or to minimize the error between the output and a desired response.

Although an adaptive array potentially yields an order of magnitude performance improvement for a missile seeker, there is an associated development risk. The complexity, cost, and transient characteristics of an adaptive array antenna suggest that practical missile implementation should be addressed. Feasibility of an adaptive missile seeker antenna will be heavily dependent on the particular implementation of the antenna array, beam-forming network, adaptive signal processor, and the chosen adaptive algorithm. One could choose an adaptive antenna system of such complexity that implementation or cost would be clearly impractical or impossible in the

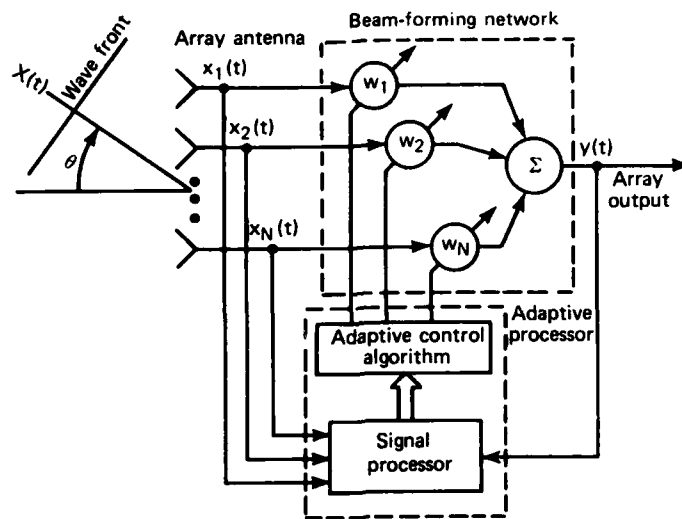


Figure 2 — Generic adaptive array antenna.

next decade. Alternatively, a simple, easily implemented adaptive missile seeker antenna could be chosen that would either respond so slowly or so inadequately that the net result would be degraded capability from a conventional fixed pattern antenna.

Thus, the first step in assessing feasibility of an adaptive missile seeker antenna is to identify and evaluate the more promising configurations and implementations relative to the requirements.

#### 4. ADAPTIVE ARRAY POTENTIAL

Before discussing specific adaptive antenna systems relative to missile seekers, consider first a simple adaptive linear array and its steady state theoretical performance against narrowband jamming. Figure 3 illustrates an adaptive linear array with  $N$  antenna elements equally spaced with separation  $d$ . For evaluating theoretical performance the antenna elements will be assumed to be omnidirectional. Each array element is weighted with a complex weight,  $w_k$  (i.e.,  $w_k = a_k + jb_k$ ) before summing to a final output,  $y(t)$ :

$$y(t) = \sum_{k=1}^N w_k x_k(t),$$

where  $x_k(t)$  is the signal impinging on the  $k$ th element. Expressing the element weights and received

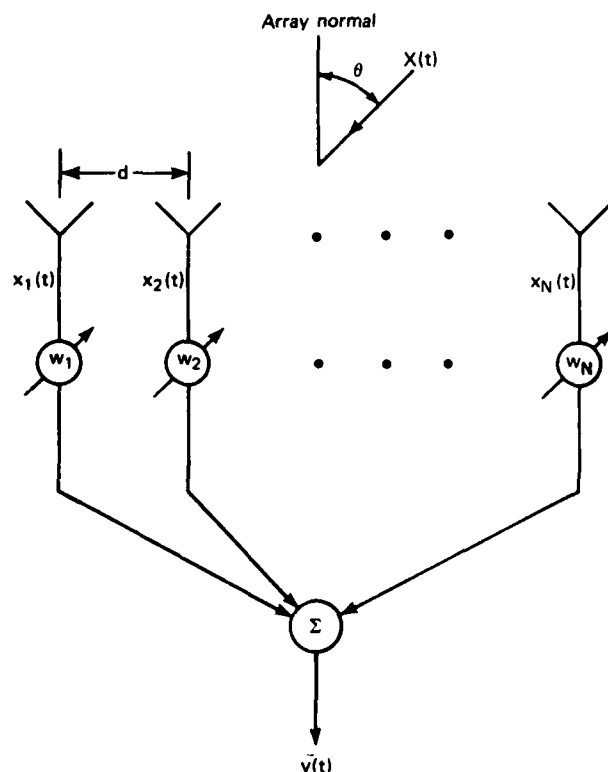


Figure 3 — Adaptive linear array.

signals as column vectors (in bold face type), the above equation can be expressed in matrix notation as

$$y(t) = \mathbf{w}' \mathbf{x}(t),$$

where

$$\mathbf{w}' = [w_1, w_2, \dots, w_N],$$

$$\mathbf{x}'(t) = [x_1(t), x_2(t), \dots, x_N(t)],$$

and  $T$  denotes transpose. The signal vector  $\mathbf{x}(t)$  consists of a desired signal vector  $\mathbf{s}(t)$  and a noise plus interference vector  $\mathbf{n}(t)$  such that

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{n}(t),$$

where

$$\mathbf{s}'(t) = [s_1(t), s_2(t), \dots, s_N(t)],$$

and

$$\mathbf{n}'(t) = [n_1(t), n_2(t), \dots, n_N(t)].$$

The signal vector  $\mathbf{s}(t)$  is assumed to originate from a single source at angle  $\theta_0$  from array normal with signal power  $S$  and radian frequency  $\omega_0$ . At the  $k$ th array element and phase referenced to the array center the desired signal takes the form

$$s_k(t) = s(t) \exp[j(2k - N - 1) \psi_0],$$

$$k = 1, 2, \dots, N$$

where

$$s(t) = \sqrt{S} \exp(j\omega_0 t),$$

$$\psi_0 = (\pi d / \lambda) \sin \theta_0,$$

$$\lambda = 2\pi c / \omega_0,$$

and  $c$  = speed of light. The noise signal vector  $\mathbf{n}(t)$  consists of  $M$  directional narrowband noise signals (jammers) and quiescent thermal noise. The composite noise signal at the  $k$ th element is

$$n_k(t) = q_k(t) + \sum_{m=1}^M J_m(t) \exp[j(2k-N-1)\psi_m]$$

$$k = 1, 2, \dots, N$$

where  $q_k(t)$  is the quiescent thermal noise component,  $J_m(t)$  is the  $m$ th directional noise component originating from direction  $\theta_m$  relative to array normal, and

$$\psi_m = (\pi d/\lambda) \sin \theta_m$$

$$\mathbf{R}_{nn} = E[(J_m(t))^2] \begin{bmatrix} 1 & \exp(j2\psi_m) & \exp(j4\psi_m) & \dots & \dots & \dots \\ \exp(-j2\psi_m) & 1 & \exp(j2\psi_m) & \dots & \dots & \dots \\ \exp(-j4\psi_m) & \exp(-j2\psi_m) & 1 & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & 1 \end{bmatrix}$$

The noise signal vector is assumed to be independent of the desired signal and the component noise signals are uncorrelated with each other. The correlation matrices needed to find the optimum element weights are defined below, where  $E[\ ]$  denotes expected operation and  $*$  denotes complex conjugate:

Desired signal correlation matrix:

$$\mathbf{R}_{ss} \triangleq E[\mathbf{s}^*(t)\mathbf{s}^T(t)]$$

Noise plus interference correlation matrix:

$$\mathbf{R}_{nn} \triangleq E[\mathbf{n}^*(t)\mathbf{n}^T(t)]$$

Received signal correlation matrix:

$$\mathbf{R}_{xx} \triangleq E[\mathbf{x}^*(t)\mathbf{x}^T(t)] = \mathbf{R}_{ss} + \mathbf{R}_{nn}$$

Received and desired signal cross-correlation vector:

$$\mathbf{r}_{sx} \triangleq E[\mathbf{x}^*(t)\mathbf{s}^T(t)] = \mathbf{S}\mathbf{v}^*$$

where  $\mathbf{v}^T = [v_1, v_2, \dots, v_N]$  (array propagation vector) and  $v_k = \exp[j(2k-N-1)\psi_0]$ . Since the noise signals are assumed to be mutually uncorrelated, it follows that

$$\mathbf{R}_{nn} = \mathbf{R}_{qq} + \sum_{m=1}^M \mathbf{R}_{nnm}$$

where  $\mathbf{R}_{qq}$  takes the form

$$\mathbf{R}_{qq} = \begin{bmatrix} E[q_1^2] & 0 & 0 & \dots & 0 \\ 0 & E[q_2^2] & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & E[q_n^2] \end{bmatrix}$$

and

$\mathbf{R}_{nn}$  is generally a positive definite Hermetian matrix.

## Signal-To-Interference-Plus-Noise Ratio Maximization

The signal-to-interference-plus-noise ratio is maximized when the array element weight vector is of the form

$$\mathbf{w}_{opt} = \alpha \mathbf{R}_{nn}^{-1} \mathbf{v}^*$$

where  $\alpha$  is a scalar constant and  $^{-1}$  denotes matrix inverse.<sup>4</sup> The optimal S/IN is then

$$S/IN_{opt} = \mathbf{s}^T \mathbf{R}_{nn}^{-1} \mathbf{s}^*$$

There are several different measures of signal reception performance one could use to adapt an array. However, the other popular performance measures of least mean square error (LMS), maximum likelihood (ML) and minimum noise variance (MV) all result in S/IN values identical to that above, under appropriate conditions.

The theoretical S/IN performance of an adaptive array can be evaluated using the above equations. The theoretical S/IN of an adaptive array is plotted in Fig. 4 versus jammer position for one, two, and four jammers. The adaptive array consists of 8 omnidirectional elements spaced  $\lambda/2$  apart. Jammers one, two, three, and four are positioned at  $+\theta$ ,  $-\theta$ ,  $+3\theta$ , and  $-3\theta$ , respectively, where  $\theta$  is the angle from array normal. Each jammer produces a 20 dB jammer-to-thermal-noise ratio in each element channel. A desired signal is in the array normal direction and yields 6 dB signal-to-thermal-noise ratio in each of the eight channels.

As expected, the S/IN degrades as the jammer(s) are positioned closer (in angular direction) to the desired signal and as the number of jammers is increased. However, when there are no jammers positioned within an angle of  $\pm \arcsin(\lambda/Nd)$  from array normal, the S/IN approaches the thermal-noise-limited value of 15 dB nearly independent of the number of jammers. This corresponds to jammers being positioned beyond the null-to-null beamwidth of an array with uniform element weights of unity. The near-thermal-noise-limited S/IN holds true as long as the number of jammers in the sidelobe region is one less than the number of adaptive channels. The effect of reduced S/IN for closely positioned jammers is caused primarily by a reduction in main beam gain as pattern nulls are placed in directions corresponding to main beam jammers. This fundamental limitation

on adaptive array S/IN results from the limited resolution of the array. Array resolution is directly related to its size in wavelengths,  $Nd/\lambda$ .

The relative performance of an adaptive array compared to the performance of a conventional array is of primary concern. S/IN in a four-jammer environment is plotted in Fig. 5 versus jammer positions for an adaptive array, a uniform array, and a Dolph-Chebyshev array. The Dolph-Chebyshev array is representative of a low sidelobe antenna. The array weights are such that, in this example, the Dolph-Chebyshev array exhibits a constant radiation intensity pattern with peak sidelobes 30 dB below the main beam peak. The uniform array has elements equally weighted to unity. When directional interference is not present (i.e., only thermal noise is present in each channel) a uniform array yields the maximum S/IN. The constant pattern of a uniform array has a peak sidelobe response 13 dB below the main beam peak. An adaptive array pattern of course varies with the interference environment to yield the maximum S/IN.

The jammers used in the S/IN computations of Fig. 5 are spaced uniformly and symmetrically about the array normal at angles of  $+\theta$ ,  $-\theta$ ,  $+3\theta$ , and  $-3\theta$ , with relative power levels equal to that of Fig. 4. The improvement in S/IN for the adaptive array is significant, especially when the jammers are closely spaced. The improvement is 10 dB or more relative to both conventional arrays for  $(Nd/\lambda) \sin \theta$  values

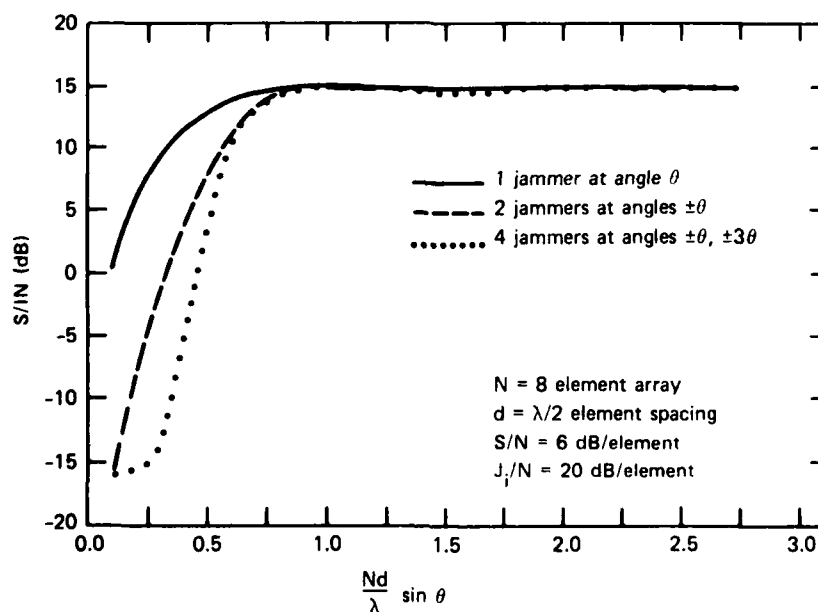


Figure 4 — Theoretical adaptive array performance.



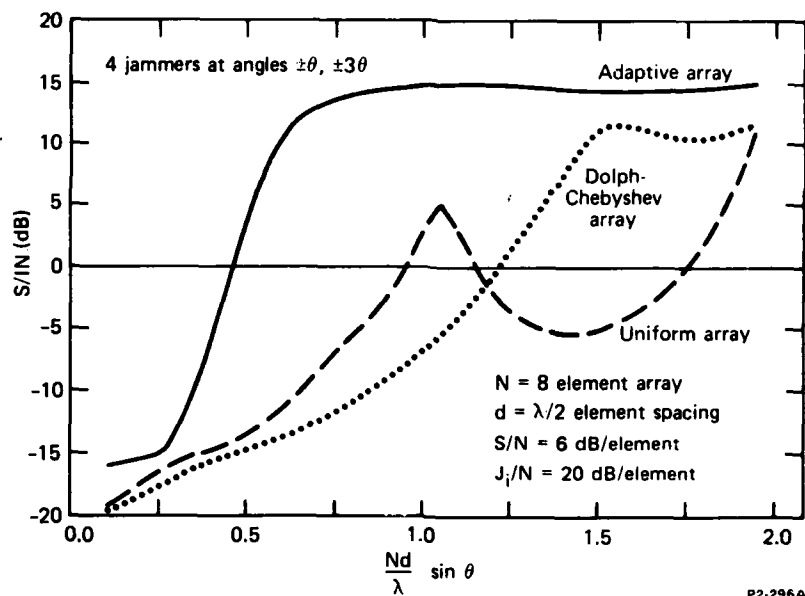


Figure 5 — Theoretical S/I/N performance comparison between an adaptive array, a Dolph-Chebyshev array, and a uniform array.

ranging from 0.36 to 1.31, or roughly an angular extent equivalent to the 3 dB beamwidth of the uniform array. Less, but significant, improvement in S/I/N is indicated over all other jammer positions. The jammers used in the computation are of moderate power. Theoretically, the improvement in S/I/N relative to a conventional array increases with jammer intensity. Figure 5 illustrates, as one would expect, the considerable jamming suppression afforded by the Dolph-Chebyshev array (i.e., low sidelobe array) when the jammers are positioned in the sidelobes. However, the performance is still not as good as the adaptive array and is significantly worse when one or more jammers is in the angular region of the main beam.

An adaptive array pattern is plotted in Fig. 6 along with patterns of the Dolph-Chebyshev and uniform arrays for a fixed four-jammer environment. The four jammers are located at angles of  $+\theta$ ,  $-\theta$ ,  $+3\theta$ , and  $-3\theta$ , respectively, as indicated in Fig. 6, where  $\theta = \arcsin(0.7\lambda/Nd) = 10.1^\circ$  is chosen to place jammers within the main beam. Figure 6 illustrates a 2 dB reduction in adaptive array gain relative to the uniform array at the main beam peak that results from placing pattern nulls within the main beam region. The pattern nulls of the adaptive array correspond to the positions of the jammers. The adaptive array sidelobes are quite high in this example but are of little concern since they are steered away from the jammers. (If, in an operational system, such increased sidelobe levels cause reception of intolerable clutter,

multipath signals, or other undesired signals, the adaptive array would have to be designed to compensate for these signals while simultaneously suppressing interference.) Even though the adaptive array pattern does not appear desirable at first glance, it in fact yields a S/I/N improvement of over 20 dB relative to the conventional arrays for this jamming environment.

Another way to illustrate the relative performance of an adaptive array antenna is to fix a jammer spacing and compute the S/I/N for a varying jamming centroid but constant desired signal direction. This corresponds to physically moving an array such that the array normal scans past a fixed geometry of jammers. In missile seeker operation, an array antenna typically is physically scanned to search for a target. Figure 7 is a plot of S/I/N versus jamming centroid for each of the three arrays. The four jammers are uniformly spaced every  $20.2^\circ$  ( $2 \arcsin(0.7\lambda/Nd)$ ). Figure 7 illustrates the main beam jamming susceptibility of each array. It is apparent that the adaptive array offers significantly improved performance regardless of jamming centroid. However, even an adaptive array cannot suppress interference that occurs in a direction equal to that of the desired signal unless more is known about the nature of the jamming and the desired signal. Of ultimate concern, however, is the probability of jamming effectiveness when searching for a target located somewhere in a given angular region. If for example, S/I/N degrada-

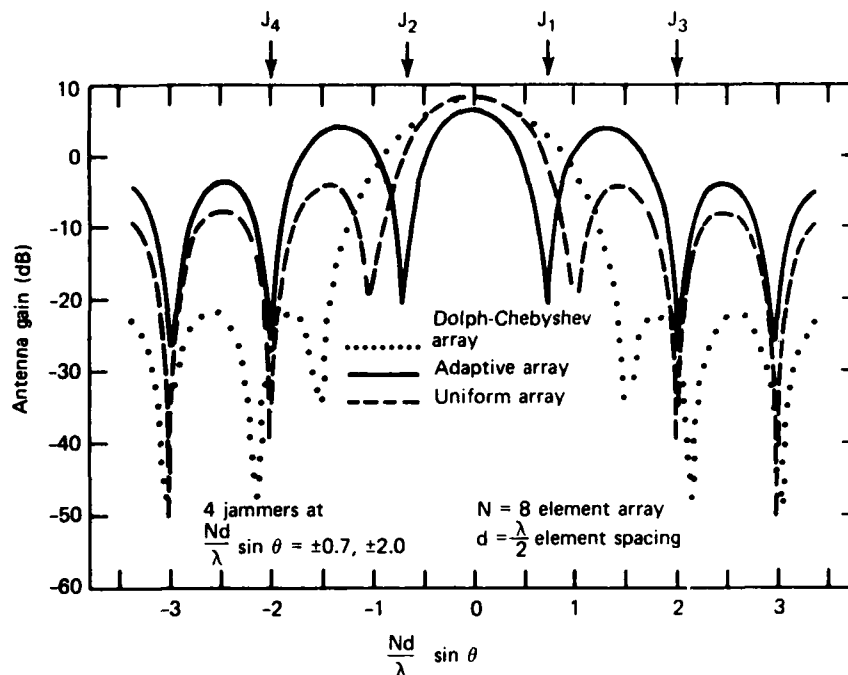


Figure 6 — Adaptive array, Dolph-Chebyshev array, and uniform array antenna patterns in jamming.

tion of 10 dB can be tolerated relative to quiescent conditions (i.e., a no-jamming S/IN of 15 dB), with what probability will the jamming inhibit detection of a target located randomly in an angular region of  $\pm 43^\circ$  ( $(Nd/\lambda) \sin \theta = \pm 3$ )? The Dolph-Chebyshev and uniform arrays are inhibited from target detection with a probability of 1. The adaptive array is inhibited from target detection only 23% of the time for this jamming environment.

### Adaptive Angle Estimation

In addition to maximizing the signal reception performance of a system, an adaptive array can be used to optimize the estimate of the direction of arrival of a desired signal. Angle estimation with a linear array requires the formation of two beams. A sum beam generally corresponds to a beam that is optimized for signal reception with element weights

$$\mathbf{w}_s = \mathbf{R}_{nn}^{-1} \mathbf{b}^*(\theta_p),$$

such that  $y_s(t) = \mathbf{w}_s^T \mathbf{x}(t)$ . The beam-steering vec-

tor,  $\mathbf{b}^*(\theta_p)$ , is matched to a source originating from angle  $\theta_p$ . A difference beam is configured to optimize angle estimation when used in conjunction with the sum beam. A maximum likelihood estimate of signal direction of arrival (DOA) is derived.<sup>18,19</sup> When using an approximation of the maximum likelihood estimate, the difference beam weights take the form

$$\mathbf{w}_d = \mathbf{R}_{nn}^{-1} \mathbf{b}'^*(\theta_p),$$

where  $\mathbf{b}'^*(\theta_p)$  is the derivative of  $\mathbf{b}^*(\theta)$  with respect to  $\theta$  evaluated at  $\theta_p$ . An approximate maximum likelihood estimate can be expressed as

$$\mathbf{R}_c \left[ \frac{\mathbf{w}_d^T \mathbf{x}(t)}{\mathbf{w}_s^T \mathbf{x}(t)} \right] = \mathbf{R}_c \left[ \frac{\mathbf{w}_d^T \mathbf{b}(\theta)}{\mathbf{w}_s^T \mathbf{b}(\theta)} \right] \bigg|_{\theta = \hat{\theta}}$$

<sup>18</sup>R. C. Davis, L. E. Brennan, and I. S. Reed, "Angle Estimation with Adaptive Arrays in External Noise Fields," *IEEE Trans. Aerosp. and Electron. Sys.*, AES-12, pp. 179-186 (Mar 1976).

<sup>19</sup>R. N. Adams, L. L. Horowitz, and K. D. Senne, "Adaptive Main-Beam Nulling for Narrow-Beam Antenna Arrays," *IEEE Trans. Aerosp. and Electron. Sys.*, AES-16, pp. 509-516 (Jul 1980).

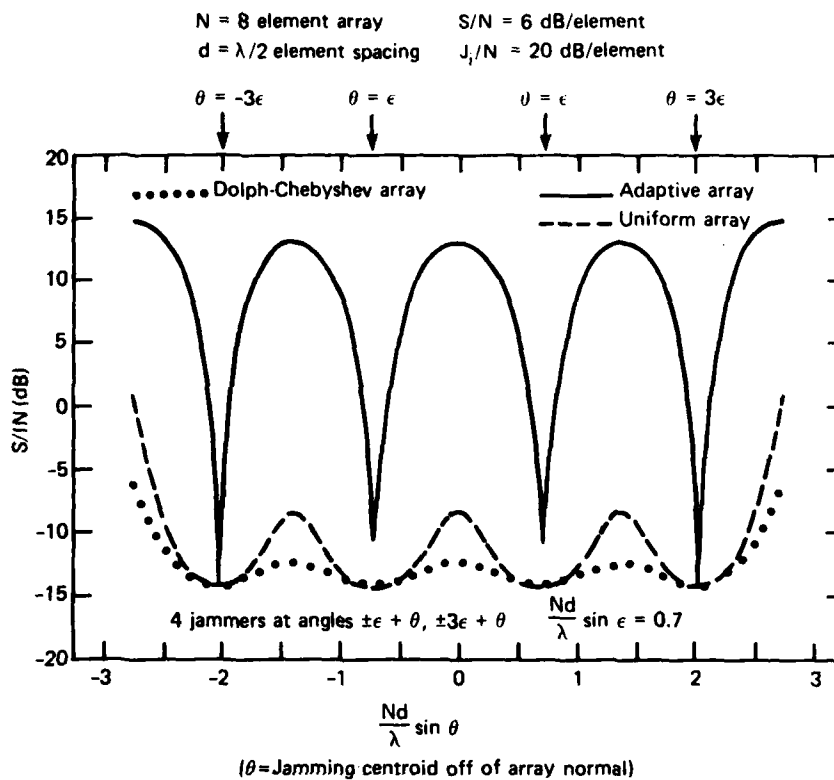


Figure 7 — Theoretical adaptive and nonadaptive array S/I/N for uniformly spaced jammers offset from array normal.

The value of  $\theta$  that satisfies the above equality is the estimated DOA of the desired signal,  $\hat{\theta}$ . The above formulation is similar to standard monopulse angle estimation. In fact, when there is no jamming present the maximum likelihood estimate reduces to standard monopulse processing.

Angle estimation with an adaptive array is particularly attractive when one or more jammers are in the array main beams. When jammed in the main beam a standard (nonadaptive) monopulse system can be captured by the interfering signal with the resulting angle estimates correspond to the interferer instead of the desired signal.<sup>20</sup> The effect of main beam jamming on the adaptive angle estimator is to increase the bias and rms errors. However, the desired source can still be tracked well within the main beam.<sup>18</sup> When jamming enters the antenna sidelobes, adaptive angle estimates based on the maximum likelihood criterion approach the accuracy of standard monopulse angle estimation without jamming.

<sup>20</sup>I. Kanter, "The Effect of Jamming on Monopulse Accuracy," *IEEE Trans. Aerosp. Electron. Sys.*, AES-15, pp. 738-741 (Sep 1979).

## Performance Limitations

Adaptive antennas offer a potential for improving missile seeker performance to a degree unmatched by other techniques. However, one should note that the previous S/I/N calculations are theoretical and confined to well-defined jamming environments. In practice, the environment and adaptive array realization is not so straightforward.<sup>21-23</sup> Signal reception and angle estimation performance can be limited by the signal environment characteristics and adaptive array system imperfections. The limiting effects of the signal environment include desired signal bandwidth, clutter, multipath propagation, and "smart" jammer techniques. The adaptive array system limi-

<sup>21</sup>J. T. Mayhan, "Nulling Limitations for a Multiple-Beam Antenna," *IEEE Trans. Ant. Propag.* AP-24, pp. 769-779 (Nov 1976).

<sup>22</sup>R. A. Monzingo and T. W. Miller, *Introduction to Adaptive Arrays*, John Wiley & Sons, (1980).

<sup>23</sup>J. T. Mayhan and F. W. Floyd, Jr., "Factors Affecting the Performance of Adaptive Antenna Systems," *Proc. 1980 Adaptive Ant. Symp.*, Rome Air Development Center, RADCR-80-378, Vol. I, pp. 154-179 (Dec 1980).

tations can be attributed to errors in the antenna array, adaptive processor, and beam-forming network.

Such limitations are typically assessed in terms of cancellation performance, which is defined as

$$C = (J/n)_a / (J/n)_b,$$

where  $(J/n)_b$  is the jamming-to-thermal-noise ratio before adaption and  $(J/n)_a$  is the jamming-to-thermal-noise ratio after adaption. The antenna pattern assumed when computing  $(J/n)_b$  is usually that of a single element of the array (i.e., the array output with all but one of the elements disabled). The cancellation performance of an adaptive array describes how well a jammer can be suppressed and implies the achievable S/IN and angle estimation performance for particular interference environments.

A potentially dominant limitation imposed by wideband jamming is antenna array disposition (i.e., antenna frequency dependence). Since there is a progressive propagation time delay across the array when signals arrive from directions other than the array normal, the relative frequency response of the array elements can differ depending on the signal location. For complex weighted arrays (frequency independent magnitude and phase weighting) array dispersion can result in incomplete jammer cancellation. Mayhan<sup>23</sup> shows that array dispersion limits adaptive cancellations to

$$C = k_1 [2\pi(D/\lambda)FBW \sin\theta_j]^2$$

where  $D/\lambda$  = array extent in wavelengths,

FBW = fractional bandwidth of interference,  
 $B/f_0$ ,

$B$  = interference bandwidth,

$f_0$  = center frequency, and

$\theta_j$  = interference angle relative to array normal.

The constant  $k_1$  depends somewhat on the type of array and configuration, but is nominally 0.0015. Thus, a 1% fractional bandwidth and 45° interference angle limits cancellation to about -30 dB for an array with a  $D/\lambda$  of 20.

It is more difficult to generalize the degradation caused by multipath propagation, since it has a complicated dependence on geometry, physical surround-

ings, and array characteristics.<sup>24</sup> Multipath propagation induces cancellation degradation because two or more correlated signals impinge on the array at different angles and with relative delays. The signals add with relative phase shifts that vary with frequency. Simple complex weighting of the signals can result in incomplete cancellation. Thus, multipath propagation can be a serious problem with an adaptive array when the reflected signals have signal powers on the order of the direct path signal and significantly different angle of arrivals.

An adaptive array can be designed to compensate for many of the limitations potentially caused by the signal environment.<sup>25-28</sup> However, the compensation circuitry adds considerably to the complexity of the adaptive array. Wideband jammers and multipath propagation can be compensated for by introducing tapped delay lines with adaptable weights for each tap (i.e., frequency-dependent weighting) in place of the single (complex) adaptive weight for each element. The intertap delays needed to compensate for wideband jamming and multipath propagation are a fraction of the inverse signal bandwidth. Severe multipath propagation can necessitate a 5 tap transversal filter (tapped delay line) for compensation.<sup>22</sup>

Although adaptive arrays can be explicitly designed to suppress jammers, they can be susceptible to jammers that exploit the array characteristics. These "smart" jammers attempt to either keep the array from converging to its optimal pattern with periodic waveforms or introduce interfering signals in time, frequency, or polarization that go unsensed by the adaptive processor. Pulsed and periodically modulated jammers can be successfully accommodated by adaptive arrays.<sup>29-30</sup> To be effective, a pulsed jammer must optimize its power, pulse repetition frequency, and pulse width relative to the internal adaptive array

<sup>24</sup>A. M. Vural, "Effects of Perturbations on the Performance of Optimum/Adaptive Arrays," *IEEE Trans. Aerosp. Electron. Sys.* **AES-15**, pp. 76-87 (Jan 1979).

<sup>25</sup>J. T. Mayhan, A. J. Simmons, and W. C. Cummings, "Wide-Band Adaptive Antenna Nulling Using Tapped Delay Lines," *IEEE Trans. Ant. Propag.* **AP-29**, pp. 923-935 (Nov 1981).

<sup>26</sup>W. E. Rodgers and R. T. Compton, Jr., "Adaptive Array Bandwidth with Tapped Delay-Line Processing," *IEEE Trans. Aerosp. Electron. Sys.*, **AES-15**, pp. 2-28 (Nov 1979).

<sup>27</sup>K. Takao and K. Komiyama, "An Adaptive Antenna for Rejection of Wideband Interference," *IEEE Trans. Aerosp. Electron. Sys.* **AES-16**, pp. 452-459 (Jul 1980).

<sup>28</sup>J. T. Mayhan, "Same Techniques for Evaluating the Bandwidth Characteristics of Adaptive Nulling Systems," *IEEE Trans. Ant. Propag.*, **AES-15**, pp. 363-373 (May 1979).

<sup>29</sup>R. T. Compton, Jr., "The Effect of a Pulsed Interference Signal on an Adaptive Array," *IEEE Trans. Aerosp. and Electron. Syst.* **AES-18**, pp. 297-309 (May 1982).

<sup>30</sup>R. T. Compton, Jr., "Advanced Adaptive Antenna Techniques," Ohio State University ElectroScience Laboratory, ESI 712684-9 (Apr 1981).

responses and physical geometry between the array and jammer. Adaptive array design practices that leave little opportunity for significant jammer energy to go unsensed will reduce the probability of successful jamming.

Limitations of adaptive array cancellation performance can be dominated by system component errors. The antenna array is subject to element placement inaccuracies and feed system errors. The adaptive processor must compute accurately the estimated environment and array weight values from the array outputs and is thus sensitive to channel tracking errors, DC offsets, quantization, in-phase quadrature channel (I/Q) imbalance, and dynamic range.<sup>31-35</sup> The beam-forming network introduces cancellations limits via feedthrough in weighting circuits, quantization, and I/Q imbalance.<sup>23,36</sup>

A potentially dominant affect, uncorrelated channel tracking errors, can be expressed as

$$\sigma^2 = \sigma_a^2 + \sigma_\phi^2,$$

where  $\sigma_a$  = rms channel amplitude mismatch (zero mean) and  $\sigma_\phi$  = rms channel phase mismatch (zero mean).

Adaptive array cancellation is directly related to channel tracking errors such that

$$C \approx \sigma^2.$$

Thus -30 dB cancellation requires channel amplitude and phase tracking within 0.2 dB and 1.3°, respectively. Prototype arrays have achieved this kind of accuracy and better.

Mayhan<sup>23</sup> identifies weight feedthrough as a potentially dominant cancellation limitation. Correlated weight feedthrough is a frequency-dependent signal feedthrough independent of weight-control voltage. The cancellation limitation is established as

$$C \leq N^2 \gamma_0^2 (\pi B \tau / \sqrt{3})^2,$$

where  $N$  = Number of adaptive channels,  $\gamma_0$  = magnitude of feedthrough component, and  $\tau$  = delay of feedthrough path relative to weighted path. Of significance is the  $N^2$  dependence of cancellation on feedthrough. Thus multichannel systems could conceivably experience severe cancellation limitations as a result of weight feedthrough.

<sup>31</sup> L. E. Brennan, E. L. Pugh, and I. S. Reed, "Control Loop Noise in Adaptive Array Antennas," *IEEE Trans. Aerosp. and Electron. Syst.*, AES-17, pp. 254-262 (Mar 1971).

<sup>32</sup> R. Nitzberg, "Canceler Performance Degradation Due to Estimation Noise," *IEEE Trans. Aerosp. and Electron. Syst.*, pp. 685-692 (Sep 1981).

<sup>33</sup> R. T. Compton, Jr., "The Effect of Random Steering Vector Errors in the Applebaum Adaptive Array," *IEEE Trans. Aerosp. Electron. Syst.*, pp. 685-692 (Sep 1981).

<sup>34</sup> R. T. Compton, Jr., "Pointing Accuracy and Dynamic Range in a Steered Beam Adaptive Array," *IEEE Trans. Aerosp. Electron. Syst.*, AES-16, pp. 280-287 (May 1980).

<sup>35</sup> R. T. Compton, "The Effect of Differential Time Delays in the LMS Feedback," *IEEE Trans. Aerosp. Electron. Syst.*, AES-17, pp. 222-228 (Mar 1981).

<sup>36</sup> J. D. R. Kramer, "Adaptive Antenna Array Processing: A Study of Weight Error Effects," *Case Studies in Advanced Signal Processing*, IEEE Conf. Publ. 180, pp. 184-189 (1979).

## 5. ASSUMPTIONS AND REQUIREMENTS

To establish some general requirements for an adaptive array, a generic high performance air intercept missile will be assumed. The adaptive array requirements will be more severe for this missile than for most others. Closing velocities could conceivably approach 10,000 ft/s. Typically, an air intercept missile will operate at a frequency between 7 and 40 GHz. A Ku-band (12.0 to 18.0 GHz) missile seeker will be assumed. A circular aperture antenna nominally 1 ft in diameter is assumed. The missile searches for a target located somewhere in a given spatial volume. Homing begins once a target is detected and acquired. Typically, Doppler processing is used to improve target detection probability and terminal guidance performance.

The primary objective of an adaptive array, as implemented on a missile seeker, will be the suppression of intentional interference. Specifically, the adaptive array should be capable of placing pattern nulls on multiple jammers occurring in the angular region of the antenna main beam and sidelobes. Although an adaptive array could be designed to compensate for cross-polarization interference and clutter, the investigation will be limited to co-polarized jamming. The significant jammer energy will be confined to a narrow band, since the instantaneous operating bandwidth of the missile will be relatively narrow (1% fractional bandwidth or less).

Current homing missiles typically employ monopulse processing to determine target angular position. A sum beam ( $\Sigma$ ) and two difference beams in azimuth and elevation ( $\Delta_a$  and  $\Delta_e$ , respectively) are formed from the same aperture and are subsequently processed to estimate target angle relative to antenna boresight. An adaptive missile seeker antenna is required to form three such beams. During target search and acquisition, the sum beam will be adapted to maximize the S/IN and thus ensure maximum probability of target detection. When homing, the three beams will be independently formed to jointly optimize target angle estimation.

An adaptive missile seeker array should be both cost-effective and practical to implement in the next decade. Adaptive array costs should be commensurate with other missile subsystem costs. The size must be limited to the space available for a conventional antenna and additional processing electronics. Because of the current efforts in very high speed integrated circuits (VHSIC), signal processors capable of

20 to 50 million complex operations per second and sized for missiles will be available in the next few years.<sup>37-44</sup> Signal processors capable of over 200 million complex operations per second are forecast for the 1990's. Therefore, an adaptive antenna processor should require less than 200 million complex operations per second if it is to be implemented in the next decade.

Since an adaptive array must first sense and estimate the environment and then adjust antenna characteristics, there is a delay between environment changes and realization of the optimum antenna response. Usually the environment as sensed by the seeker antenna changes most rapidly when the antenna is scanned during target search. The time spent dwelling on a target cell can be driven by the antenna mechanical scan rate, target correlation time, target closing velocity, etc. Typically, a mechanically steered antenna scans at 150°/s or less. This establishes the target dwell time for an antenna of a given beamwidth. Electronically steered antennas can be scanned more rapidly and allow the target dwell time to be chosen for improved target detectability. Since air target correlation times can be in the tens of milliseconds, the smaller target dwell time is often established by uncompensated closing velocity. Uncompensated closing velocity is that component of missile-to-target closing velocity that is not adjusted for in the missile (e.g., relative range gate position adjustment).

When the criteria is target traversal through a range gate, the target dwell time will be

$$T_d \leq c\tau_r \delta_r / 2V_{cl}$$

<sup>37</sup> "Technical Survey: Very High Speed Integrated Circuits," *Aviat. Week & Space Technol.*, pp. 48-83 (16 Feb 1981).

<sup>38</sup> J. Martin, "Very High Speed Integrated Circuits - Into the Second Generation, Part 1," *Mil. Electron.*, 7, p. 52 (Dec 1981).

<sup>39</sup> J. Martin, "Very High Speed Integrated Circuits - Into the Second Generation, Part 2," *Mil. Electron.*, 8, p. 60 (Jan 1982).

<sup>40</sup> J. Martin, "Very High Speed Integrated Circuits - Into the Second Generation, Part 3," *Mil. Electron.*, 8, p. 83 (Feb 1982).

<sup>41</sup> J. Martin, "Very High Speed Integrated Circuits - Into the Second Generation, Part 4," *Mil. Electron.*, 8, p. 40 (Mar 1982).

<sup>42</sup> "VHSIC Processors Take Shape," *Def. Electron.*, 14, pp. 33-38 (Aug 1982).

<sup>43</sup> E. Brookner, "Trends in Radar Signal Processing," *Microwave J.*, 25, pp. 20-39 (Oct 1982).

<sup>44</sup> L. W. Sumney, "VHSIC: A Status Report," *IEEE Spectrum*, 19, pp. 34-39 (Dec 1982).

where  $c$  = speed of light,  $\tau_c$  = effective pulse width (i.e., range gate duration),  $\delta_r$  = permissible fraction of range gate traversal, and  $V_u$  = maximum uncompensated closing velocity. The target dwell time should be the maximum of the above equation, to allow for maximum adaptive array convergence time. A maximum uncompensated closing velocity of 5000 ft/s and permissible range gate traversal of 0.5 results in a target dwell time of 50,000  $\tau_c$ . The permissible convergence time of an adaptive array is specified as 20% of the target dwell time, or

$$t_c = 10,000 \tau_c,$$

where  $t_c$  = maximum time to adaptive array convergence. The adaptive array must converge within a fraction of target dwell time to ensure that the S/IN is maximum for this limited target detection opportunity. Independent samples of the environment can occur every  $\tau_c$  seconds. Therefore, an adaptive missile seeker antenna is required to converge in less than 10,000 independent samples. The more restrictive target dwell time based on closing velocity has been

chosen to establish the adaptive array convergence requirement. This requirement could be relaxed for a number of missiles.

The adaptive weight control for the seeker antenna should be implemented to minimize distortion of the target return signal. Improperly designed adaptive arrays can cause severe signal distortion even though the jammer suppression requirements are achieved. This situation should be avoided. In particular, the effective antenna gain in the target direction must remain constant during the dwell time required for target Doppler measurement. The Doppler dwell time is equal to  $1/f_r$ , where  $f_r$  is the Doppler resolution bandwidth. Uncompensated antenna gain changes on the target introduces additional noise into the Doppler measurement.

The general requirements established for an air intercept missile are summarized in Table 1. The requirements are intended to be more severe than the majority of missions. These requirements will be used to establish acceptable adaptive array techniques and configurations.

Table 1 — Adaptive missile seeker antenna requirements and assumptions.

Assumptions	
Seeker functions	Search, acquisition, homing
Operating frequency	15 GHz (Ku band)
Antenna diameter	1 ft
Targets	Air
Maximum uncompensated closing velocity	5000 ft/s
Signal processing	Doppler filtering, target detection, target angle estimation
1990's processing capability	200 million complex operations per second or greater
Jamming	Co-polarized, 1% or less fractional bandwidth
Requirements	
Domain of adaptivity	Pattern nulling in main beam and sidelobes
Performance criteria	Target detection (S/IN) and Target angle estimation (maximum likelihood)
Adaptively formed beams	Sum beam, azimuth difference beam, elevation difference beam
Number of jammers accommodated	Multiple
Maximum time of converge	$1 \times 10^4 \tau_c$ ( $\tau_c$ = minimum independent sampling period)
Pattern adaption limitations	Minimal target signal distortion
Physical limitations	Size, weight, power consumption within missile constraints
Cost	Commensurate with other missile subsystems

## 6. ARRAY ANTENNAS

There are a variety of antennas that can satisfy the required radiation characteristics for missile guidance applications. The prime objective of the antenna is to enhance the desired signal reception and to reject interfering signals. An array of sensor elements, when combined with an adaptive processor, can improve overall signal reception performance in an environment having a number of interference sources. To achieve this potential it is necessary to understand the nature of the signals to be received and the properties of an array of sensor elements.

The principal system elements in an adaptive antenna system are the sensor array, the beam-forming network, and the adaptive processor. The array itself consists of a number of sensors designed to receive or transmit a desired signal. The sensor elements are arranged to give adequate gain, resolution, sidelobe levels, and beam-forming flexibility over a given spatial region. The selection of the sensor elements and their physical arrangement places fundamental limitations on the ultimate capability of the adaptive array system. The output of each element is directed to the pattern-forming network where it is multiplied by a complex weight (phase and amplitude) and then summed with all other weighted sensor element outputs to form the overall adaptive array output signal. The weight values within the pattern-forming network determine both the amplitude and phase distribution over the aperture and the overall array beam pattern.

### Fully Adaptive Arrays

A fully adaptive array configuration offers the maximum control of the antenna pattern. Each element of the array is individually controlled to form an adaptive antenna pattern. A typical missile array antenna contains many active elements. This poses several problems. Implementing an adaptive array with many degrees of freedom (an adaptive processor with  $n$  degrees of freedom requires  $n + 1$  antenna inputs) significantly affects system cost, weight, and volume. Also, with a large number of adaptive weights, the computational time (time to optimize the array pattern) can be excessively long.

### Filled Array

A missile antenna diameter of 12 in. is assumed, as in Table I. At Ku band, 185 elements spaced  $\lambda/2$

would be needed to completely fill the aperture. In order to adaptively control each individual array element, 185 separate transmission lines and 185 weighting circuits would be required. Sum and difference beams would mandate separate weighting circuits, which could increase threefold the number of controls needed. A feed system and beam-forming network this large would exceed the volume and weight constraints of present missiles. Advances in active arrays, microstrip technology, and minaturization could make such a configuration possible, but beam convergence time may still be unacceptable. It is desirable to reduce the total number of elements or adaptive degrees of freedom, while maintaining as much control as possible over the antenna pattern. Reducing the array's adaptive dimensions is more likely to meet missile guidance weight and size constraints.

### Thinned Array

An approach to reducing the number of adaptive dimensions while maintaining complete control over the individual elements is array thinning.<sup>22,45,46</sup> The number of elements may be reduced to a fraction of those needed to completely fill the aperture without suffering serious pattern degradation. If the elements are reduced in a random manner the mean sidelobe level can be kept at a specific level. The drawback to this approach is a reduction in antenna gain relative to a filled array. The decreased gain is caused by the reduced number of elements. The antenna's half-power beamwidth is a function of aperture size and is thus the same for a thinned or filled aperture. Although a thinned array offers high resolution while reducing the system complexity and cost, the reduction in antenna gain and reduced control over the sidelobes makes it unsuitable as an adaptive missile seeker antenna. For example a Ku band missile antenna thinned from 185 elements to a more manageable 30 elements would result in a reduction of antenna gain of about 8 dB.

<sup>45</sup> B. D. Steinberg, *Principles of Aperture and Array System Design*, John Wiley & Sons (1976).

<sup>46</sup> R. M. Davis and J. L. Gleich, "Element Placement in Adaptive Arrays and Sidelobe Cancellers," *Proc. 1980 Adaptive Ant. Symp.*, Vol. II, Rome Air Development Center RADC-TR-80-378, pp. 49-68 (Dec 1980).



## Partially Adaptive Arrays

A partially adaptive array is one in which elements are controlled in groups or in which only certain elements are controllable.<sup>47,48</sup> Partial adaptive control of an array is potentially an effective method for reducing the adaptive dimensions of an array while still maintaining antenna gain, resolution, and desired sidelobe levels for a given aperture. There are several methods of configuring partially adaptive arrays that can be generally categorized as multiple sidelobe cancellation and adaptive subarray beamforming.

### Multiple Sidelobe Cancellers

Multiple sidelobe cancellers adapt a chosen set of  $M$  array elements (where  $M < N$  for an  $N$  element array) to form a cancellation beam. A main array output is formed by nonadaptively summing the outputs of all  $N$  array elements. The adaptively formed cancellation beam is subtracted from the main array output to cancel interference. In general, multiple sidelobe cancellers can cancel one less jammer than the number of adaptive elements. This approach to array adaptation is sensitive to the position of adaptive elements within the array. Theoretical error-free performance is best when the adaptive elements are clustered toward the edges of the array.<sup>48</sup> However, error effects may dominate array performance. In such a case, adaptive elements clustered toward the center are more suitable since they can cancel interference somewhat independently of adaptive weight errors.<sup>49</sup>

The multiple sidelobe canceller approach is not well suited for the missile seeker antenna application. The primary disadvantage of multiple sidelobe cancellation is its ineffectiveness in cancelling main beam jamming. Cancellation of main beam and close-in sidelobe jamming is a primary requirement of an adaptive missile seeker antenna. In addition, multiple sidelobe cancellers are less suited to optionally forming the multiple beams necessary for angle estimation.

### Adaptive Subarray Beamforming

Combining an array of  $N$  elements into  $M$  subarrays enables adaptation to occur at the subarray level with a significantly reduced adaptive dimensionality. Each subarray output is controlled with an adaptive

weight before summing to the final output.  $M-1$  separate jammers can be suppressed in this manner. The transformation from  $N$  elements to  $M$  subarrays can be accomplished in a variety of ways. A straightforward approach is to combine contiguous elements into physically separated subarrays. Since the subarrays are typically separated by multiple wavelengths, gating lobes can result. The gating lobes are reduced somewhat by the subarray pattern. Once established, however, the grating lobes cannot be reduced further by the adaptive weighting.

A particularly attractive subarray transformation uses the entire aperture to form several adjacent beams that are subsequently adapted. Termed a "beam-space" array, this method can provide performance approaching that of a fully adaptive array with a reduced sensitivity to errors compared to other partially adaptive techniques.<sup>47</sup> The transformation from array elements to beam-space subarrays can be achieved with a Butler matrix<sup>50,51</sup> or other beam-forming network. The disadvantage to this approach is the complexity of the beam-forming network. A Rotman lens network can be used to form multiple beams with a less complex lens structure.<sup>16,52-56</sup> Space-fed arrays<sup>57,58</sup> can also be used for beam-space adaptation, but typically require more volume than equivalent planar arrays.

Adaptivity in a principle plane can be achieved by combining the rows (or columns) of a planar array into subarrays. Two-dimensional adaptivity is possible when the rows and columns of a planar array are combined into subarrays. Such a row-column precision array (RCPAA)<sup>59</sup> requires the splitting of each

<sup>47</sup> D. J. Chapman, "Partial Adaptivity for the Large Array," *IEEE Trans. Ant. and Propag.*, **AP-24** (Sep 1976).

<sup>48</sup> D. R. Morgan, "Partially Adaptive Array Techniques," *IEEE Trans. Ant. and Propag.*, **AP-26**, No. 6, pp. 823-833, (Nov 1978).

<sup>49</sup> R. Nitzberg, "OTH Radar Aurora Clutter Rejection when Adapting a Fraction of the Array Elements," *EASCON 1976 Record*, Washington, pp. 62.A-62.D (Sep 1976).

<sup>50</sup> J. Butler, "Multiple Beam Antenna," Sanders Associates, RI 3849 (8 Jan 1960).

<sup>51</sup> R. Levy, "A High Power X-band Butler Matrix," *Proc. Mil. Microwaves 1982*, London, pp. 580-585 (Oct 1982).

<sup>52</sup> W. Rotman and R. F. Turner, "Wide Angle Lens for Line Source Applications," *IEEE Trans. on Ant. Propag.*, **AP-11**, pp. 623-632 (Nov 1963).

<sup>53</sup> D. Archer, "Lens-Fed Multiple-Beam Arrays," *Microwave J.*, pp. 37-42 (Oct 1975).

<sup>54</sup> M. S. Smith, "Design Considerations for Ruze and Rotman Lenses," *Radio Electron. Eng.*, **52**, pp. 181-187 (Apr 1982).

<sup>55</sup> D. T. Thomas, "Antenna Systems at Raytheon TSD," *IEEE Ant. Propag. Newslett.* (D. H. Schaubert, Ed.), **24**, pp. 5-8 (Oct 1982).

<sup>56</sup> J. B. L. Rao, "Multifocal Three-Dimensional Bootlace Lenses," *IEEE Trans. on Ant. Propag.*, **AP-30**, pp. 1050-1056 (Nov 1982).

<sup>57</sup> D. G. Burks, "Main-Beam Constraints for Space-Fed Adaptive Arrays," *Proc. 1980 Adaptive Ant. Symp.*, Vol. II, Rome Air Development Center, RADC-TR-80-378, pp. 69-89 (Dec 1980).

<sup>58</sup> R. J. Mailloux, "Phase Array Theory and Technology," *Proc. IEEE*, **70** (Mar 1982).

<sup>59</sup> R. W. Howells, "High Quality Array Beamforming with a Combination of Precision and Adaptivity," Syracuse University Research Corporation, SURC TN 74-150, (13 Jun 1974).

element signal into two paths to achieve both row and column summing of element outputs. This type of array has many of the advantages of a beam-space adaptive array with a somewhat less complex feed network.<sup>46</sup>

Subarray beam-forming is an attractive approach to achieving partial adaptivity (reduced adaptive dimensionality) for a missile seeker antenna. However, particular subarray methods will be more suited than others to a missile antenna application. Under error-free conditions, adaptive subarray beam-forming can approach the performance of a fully adaptive array.<sup>47</sup> However, the performance of adaptive subarray beam-formers degrades more quickly with array element errors than does a fully adaptive array. This is a result of the adaptive processor's control of array elements in groups instead of the individual element control afforded by a fully adaptive array. The reduced dimensionality of adaptive subarrays can also result in significant performance degradation when the number of jammers equals or exceeds the number of formed subarrays. The performance of a fully adaptive array is much less sensitive to the number of jammers, even when they exceed the number of elements in the array. Nevertheless, adaptive subarray beam-forming offers a potential for implementation in a missile seeker unavailable in the foreseeable future with a fully adaptive array.

### Array Pattern Formation

An array is an aperture excited only at points or in localized areas. It consists of discrete radiating elements. The simplest form of an array is one in which the elements are uniformly spaced along a straight line. If each of these elements is weighted with a complex weight, the radiation pattern is simply

$$f(u) = \sum_{n=0}^{N-1} w_n \exp[j(2\pi d/\lambda)nu],$$

where

- $N$  = number of elements in the array,
- $\lambda$  = wavelength,
- $d$  = spacing between elements,
- $u = \sin \theta$ ,
- $\theta$  = angle relative to array normal, and
- $w_n$  = complex weight for element  $n$ .

When the array consists of identical antenna ele-

ments, each with a pattern defined by  $e(u)$ , the array pattern is

$$g(u) = f(u) e(u).$$

The first term is the array pattern or array factor. It contains the geometric information of the array and pattern control information. The second term is called the element factor.<sup>58,60,61</sup> It is evident from this equation that the element pattern and array excitation determine the array radiation performance. Control of the individual elements or groups of elements makes array adaptation possible within the constraints of the element pattern and array geometry.

A planar array allows control in two orthogonal directions and is an extension of the linear array. The pattern for an idealized rectangular planar array of identical elements with separable excitation can be separated into element, row, and column factors. The resulting planar array can be expressed as

$$g(u, v) = e(u, v) f_1(v) f_2(u),$$

where

- $e(u, v)$  = element pattern,
- $f_1(v)$  = row array factor,
- $f_2(u)$  = column array factor,
- $u = \sin \theta \cos \phi$ ,
- $v = \sin \theta \sin \phi$ ,
- $\theta$  = angle from array normal, and
- $\phi$  = angle relative to an array principle axis.

A planar aperture produces a pencil beam with reasonable control of minor lobes. It has significant antenna gain and also the potential for highly flexible adaptive control.

An antenna also acts as a transformer to provide maximum transfer of energy between the power source and free space. The antenna impedance affects this transfer. In a electronically scanned array, the antenna impedance varies with scan angle. This undesired property results from positioning the elements at closely spaced, regular intervals.<sup>62</sup> The im-

<sup>60</sup>L. Stark, "Microwave Theory of Phased-Array Antennas - A Review," *Proc. IEEE*, 62 (Dec 1962).

<sup>61</sup>T. C. Cheston and J. Frank, *Array Antennas*, JHU APL TG 956 (Mar 1968).

<sup>62</sup>A. A. Oliner and G. H. Knittel, *Phased Array Antennas*, Artech House (1970).

pedance variation is due to the coupling of energy from element to element. Coupling is by radiation, from surface paths, within the feed structure, and reflections at the antenna terminal. The principal effects of mutual coupling are effective element pattern changes, gain variations during scanning, null shifting, null filling, and effective aperture taper alteration. It is desirable to reduce mutual coupling to a negligible quantity. Mutual coupling can be controlled somewhat by the array geometry and choice of element types.

### Array Elements

There are a number of radiating antenna elements that could be considered for use in an adaptive array antenna. Some of the more popular types are open-ended waveguide, dipoles, slots (waveguide or stripline) and microstrip radiators. Frequency is a determining factor in the element selection. At frequencies above X band, dipole techniques become difficult; below X band, waveguide array tend to become heavy. Stripline techniques are useful through Ku band. The power-handling capacity of the radiating elements is another important factor, with waveguide having the greatest capacity. The beamwidth of the elements must be consistent with the angle scan coverage of the array. Usually the elements are of low gain.

One of the continuing needs of future array technology is the development of truly low cost, lightweight arrays. A step in this direction is the microstrip array and feed networks. The primary reason for this interest is to take advantage of the fabrication cost achieved by etching elements and transmission lines.

### Feeds

The critical role of the feed network in an adaptive array is to provide the desired control over the amplitude and phase distribution across the array aperture.<sup>58,60,61,63</sup> There are two general types of feeds. The first of these employs transmission line techniques entirely in routing signals from the element array to the feed points. This type of feed transmits by closed paths and is called a constrained feed. The second method uses free-space propagation to spread the signal out from a terminal to the individual elements and is referred to as a space feed.

While a constrained feed allows for flexible element and subarray control, it is susceptible to producing larger amplitude errors than a space feed be-

cause of the cumulative effects of mismatches. Thus a careful design effort must be exercised to realize the capabilities of a constrained feed. Errors in the transmission path can be reduced to acceptable levels by using well-matched components.

An example of complex but useful feed is the Butler Matrix.<sup>50,51,60</sup> It employs hybrid junctions and fixed phase shifters to form simultaneous beams. This overlapping of beams is ideally carried out without interaction or intercoupling loss to reduce the antenna gain. However, the Butler Matrix requires a large number of transmission line crossovers. The result of cascading a large number of these junctions produces a large VSWR and appreciable coupling. For this reason the Butler Matrix has not been used to directly feed the elements of a large phased array but has rather been used to feed subarrays of elements.

When the number of elements required is large, the expense and complexity of a large number of transmission line components may be avoided by the use of a space feed. The amplitude control at the secondary aperture is achieved by controlling the elements of the primary aperture (or feed array). The space feed is subject to spillover loss because the illumination is not confined to the angle subtended by the array. The constrained feed, on the other hand, is subject to much more transmission loss than the space feed. The volume of space used is larger for the space feed system. The depth of the constrained feed away from the aperture may be of the order of one-tenth the aperture dimensions, whereas it will be of the order of one-half the aperture size for a space feed.

### Potential Missile Array Antennas

There are a number of array configurations that could be used for an adaptive missile seeker antenna. However, the physical constraints of missile implementation and the relatively severe requirements for antenna adaption limit the practical array candidates to only a small number. The suitability of several array configurations for the adaptive missile seeker antenna application are summarized in Table 2. The array configurations are generally categorized as (1) planar arrays, (2) hybrid arrays, (3) space-fed arrays, and (4) conformal arrays.

### Planar Array

Many current missile seekers (as well as many that are currently under development) use planar arrays with fixed element weights in accordance with the feed structure and individual elements. Although a fully adaptive planar array is desirable from a perfor-

<sup>63</sup>M. I. Skolnik, *Introduction to Radar Systems*, 1980, McGraw Hill, (1980).

**Table 2** — Adaptive array suitability to missile seekers

<i>Array type</i>	<i>Advantages</i>	<i>Disadvantages</i>	<i>1990's suitability</i>
<b>Planar</b>			
Fully adaptive	Superior performance Flexible, error tolerant	Excessive control and feed circuitry Excessive processing requirement	Not suitable <sup>1</sup>
Thinned fully adaptive	Reduced adaptive dimensionality Simple and direct element control	Reduced antenna gain Reduced sidelobe control	Not suitable
Multiple sidelobe canceller	Reduced adaptive dimensionality Simplified control	Ineffective against main beam jammers Sensitive to adaptive element locations	Not suitable
Simple subarray	Reduced adaptive dimensionality Straightforward feed structure Tailorable to processing capability	Grating lobe effects	Suitable
Beam-space (e.g., Rotman lens network)	Reduced adaptive dimensionality Superior performance Approaching fully adaptive array	Complex feed structure Potentially large volume and weight	May be suitable <sup>2</sup>
Row-column subarray	Reduced adaptive dimensionality Good performance	Moderately complex feed structure	Suitable
<b>Hybrid</b>	Reduced adaptive dimensionality High gain, simplified control	Larger volume than planar array Blockage effects with reflector Potentially heavy with lens	Suitable
<b>Space-fed</b>	Reduced adaptive dimensionality Simplified feed structure	Larger volume than planar array Lens array phase control complexity and cost	Suitable <sup>1</sup>
<b>Conformal</b>	Conforms to missile shape Radome not required	Complex geometries and control Inefficient element usage High cost	Not suitable

**Notes:**

<sup>1</sup> Development of practical active arrays with microwave integrated circuits will improve the likelihood of a fully adaptive array. However, processing capability would also have to improve beyond that forecast.

<sup>2</sup> Additional assessment of Rotman lens beamforming for missiles is needed.

<sup>3</sup> Since a hybrid array would require mechanical scanning and adaptation about a mechanical boresight, a hybrid-reflector array is better suited than the heavier hybrid-lens array.

mance perspective, the complexity of its implementation and the processing requirement are excessive for a 1990's missile seeker. Active arrays implemented with microwave integrated circuits (MIC) could change this outlook somewhat. However, the processing requirement for a fully adaptive seeker array, as will be discussed later, is beyond that forecast for the next decade. The processing requirement of a

thinned fully adaptive array could be held to acceptable levels by significantly reducing the number of array elements within the aperture. However, the resulting loss in antenna gain is not acceptable for a system where target detectability and tracking are of critical importance. A multiple sidelobe canceller is not suited for the defined application since it is ineffective in suppressing main beam jammers.

A simple subarray approach to array adaption appears suitable for a missile seeker application. Subarrays can be formed consistent with processing capability. The feed network can be kept moderately simple. Since grating lobe effects can be a problem, a careful subarray design will be needed to minimize these effects. A beam space planar array using a Butler beam-forming matrix or equivalent is probably too complex for a missile seeker. However, multiple beam formation with Rotman lenses (Bootlace lens)<sup>19,52,56,64-66</sup> may be suitable for a missile in the next decade. A detailed assessment of a Rotman lens beam-forming network is required before one could be sure of the missile application. A row-column subarray is a suitable adaptive seeker array approach. The number of adaptive dimensions is equal to the total number of rows and columns of the array and thus has greatly reduced processing requirements. The feed network is more complex than that of simple subarrays but is considerably less complex than a Butler matrix.

### Hybrid Arrays

Hybrid arrays can meet the requirements of high antenna gain and reduced adaptive dimensions.<sup>58,67</sup> A hybrid adaptive array consists of an adaptively controlled feed array that illuminates a radiating aperture (such as a reflector or lens). The elements of the feed array are controlled separately and form separate beams after transformation by the radiating aperture. Thus, limited dimension beam space adaption can be achieved with a hybrid array. The feed array is practically limited in size because of blockage effects with a reflector. Since electronic beam scanning is typically limited to  $\pm 10^\circ$ , a missile seeker hybrid array would probably be mechanically scanned with adaption occurring about mechanical boresight. A hybrid lens array can be so heavy as to complicate mechanical scanning. Although the volume requirements are greater than for a planar array, a hybrid reflector array appears to be suitable for the missile application. Blockage effects need to be analyzed carefully

to determine if resultant pattern distortion can be tolerated.

### Space-Fed Arrays

A typical space-fed array is configured with a primary feed array that illuminates a lens array.<sup>58,68</sup> Each element in the lens array collects radiated energy from the feed array, shifts the phase to properly focus and steer, and re-radiates the energy into space. The lens array transforms each feed array element into a beam whose beamwidth and gain are established primarily by the lens array characteristics. The number of elements in the larger lens array is equal to an equivalently sized conventional array (e.g., 185 elements for a 12 in. diameter Ku-band antenna). By adapting the smaller primary feed array for S/IIN maximization and optimum angle estimation, the adaptive dimensions are greatly reduced. Beam scanning can be achieved by deterministically controlling the individual lens array phase shifters. This phase control of the lens array adds to the overall complexity and cost of the array, but allows for relatively wide scan angles and circumvents the need for mechanical scanning. When mechanical scanning is not required, space within the missile radome can be used more efficiently for antenna aperture. In general, the volume requirements of a space-fed array are greater than a planar array and correspond to a hybrid array. However, a space-fed array is suitable for a missile seeker. It has the potential for effective pattern adaption with reduced adaptive dimensionality. The major drawback to this approach is the complexity and cost of the lens array phase control.

### Conformal Arrays

Placing array elements on a nonplanar surface and achieving a directional beam can be advantageous for aircraft and missile applications.<sup>58,69</sup> Major developments have been made in arrays conformal to missile and aircraft nose cones. Requirements for flush-mounted or low-profile aircraft antennas for satellite communication have been a new stimulus for the development of conformal arrays. Array elements on curved bodies point in different directions, making it necessary to turn off those elements that radiate primarily away from the desired direction of radiation. Also, the radiation pattern cannot, in general, be separated into an element factor and array factor as in planar array theory, making conformal array synthesis difficult. As a result of the geometry, the azimuth pattern of the array may change with both azimuth and elevation scan. Mutual coupling problems can be severe. Because of the different element

<sup>64</sup>S. P. Applebaum and D. J. Chapman, "Adaptive Arrays with Main Beam Constraints," *IEEE Trans. Ant. Propag.*, AP-24, pp. 650-651 (Sep 1976).

<sup>65</sup>B. Widrow and J. M. McCool, "A Comparison of Adaptive Algorithms Based on the Methods of Steepest Descent and Random Search," *IEEE Trans. Ant. Propag.*, AP-24, pp. 615-637 (Sep 1976).

<sup>66</sup>C. A. Baird and G. G. Rassweiler, "Search Algorithms for Sonobuoy Communication," *Proc. Adapt. Ant. Sys. Workshop*, March 11-13, 1974, NRL Report 7803, Vol. 1, pp. 285-303 (Sep 1974).

<sup>67</sup>J. T. Mayhan, "Adaptive Nulling with Multiple-Beam Antennas," *IEEE Trans. Ant. Propag.*, AP-26, (1978).

pointing directions on a curved surface, cross-polarization effects arise, causing the polarization vector projection to be nonaligned. In addition, there is typically a need for different collimating phase shifters for both scanning directions. Array cost is high due to the number of redundant elements required. Missile front-end geometry is such that all these problems are made worse. The complexity and cost of such an array, especially when adapted, make it an unlikely candidate for a missile seeker.

## Errors

Perturbations in the phase and amplitude distribution across the array aperture results in pattern errors.<sup>45,68</sup> In general, amplitude and phase errors are independent of each other and the mean value of each is zero. Errors are small in the typical case; otherwise, the design pattern would bear little resemblance to the actual pattern. However, antenna adap-

tivity places severe requirements on the array. Distribution errors affect the pattern characteristics by causing gain variations, increased sidelobe levels, shift in the beam position, and null filling.

Phase errors are caused by misalignment of the array elements and transmission line length errors in the feed system. Most array components are phase sensitive to frequency. Thus frequency variations in the signal passing through the array result in phase errors. Systems that use digital phase shifters quantize the phase level causing additional errors. The most severe amplitude error is caused by the failure of an element. Other causes of amplitude errors are element rotation, misalignment of the elements, and variations in the individual elements.

Mutual coupling is a dominant phenomenon affecting both amplitude and phase. Amplitude and phase also are affected by programming and control errors in the signals driving the phased array beamforming. In adaptive systems, the control signals that determine the complex weights introduce phase and amplitude errors by virtue of noise and other disturbances inherent in the control process.

<sup>68</sup>B. M. Potts, J. T. Mayhan, and A. J. Simmons, "Some Factors Affecting Angular Resolution in Adaptive Antenna," *Int. Conf. Commun.* (Jun 1981).

## 7. ADAPTIVE ALGORITHMS

The adaptive processor is central to any adaptive array. The adaptive processor automatically adjusts array element weights to optimize performance in response to the sensed environment. The adaptive processor attributes, including complexity and transient response speed, are established primarily by the adaptive algorithm. A number of adaptive algorithms have been developed, each of which has separate strengths and weaknesses relative to the overall adaptive array implementation. The features of the adaptive algorithms to be described include: random search, gradient-based, direct matrix inverse, recursive, and cascade preprocessor. The relative complexity, computational requirements, transient response speed, and error sensitivity of each type of algorithm are assessed. The algorithms are evaluated for practical adaptive processor implementation in a missile seeker relative to the established requirements.

### Operational Constraints

Before describing the adaptive algorithms, some general comments on their practical operation are in order. As mentioned previously, there are several criteria one could use to optimally adjust the response of an array. The primary criterion here is S/IN maximization. To optimize S/IN there must be a way to discriminate between desired and undesired signals. Otherwise adaptive array performance can be degraded. Since a suitable replica of the desired signal is typically not available to correlate with the received signals in a missile seeker, signal discrimination must be accomplished with filtering techniques. Filtering on the basis of known signal parameters can be accomplished in the spatial, frequency, and time domains. For the missile seeker application, the desired signal direction is assumed to be known. If

the signal is not in the current look direction, an exhaustive search of a specified volume will be made until the look direction corresponds to the actual desired signal direction. Other desired signal parameters (frequency, bandwidth, and times of arrival) are either known or can be determined. The following discussion on adaptive algorithms assumes that the desired signal has been filtered from the undesired interference prior to the adaptive processor.<sup>64</sup> These techniques, as briefly outlined below, are described in Ref. 64 as main beam constraints.

Angle domain methods effectively constrain the adaptive response in the desired look direction. These methods include the use of pilot signals, preadaptation spatial filters, and control loop spatial filters. Such methods affect the ability of the adaptive array to cancel main beam jammers, since they constrain optimal performance in the angular region of the main beam. However, cancellation of main beam jammers has been identified as a valuable objective of an adaptive missile seeker antenna. Time and frequency domain methods of algorithm constraints do permit main beam jammer cancellation. These methods rely on the ability to sense the signal environment when the desired signal is not present but interfering signals are. In the time domain, the environment would be sensed when target or clutter returns would be either significant or nonexistent. In the frequency domain, the environment would be sensed out of the desired signal band.

Frequency and time domain algorithm constraint techniques appear to be more desirable for a missile implementation. Careful assessment needs to be made, however, of the specific jamming environments and particular jammer characteristics to be sure that jammer energy is present when the environment is sensed. In some cases, the missile seeker should be specifically designed to force jammers to place energy in specific frequency bands or time slots. Improper use of these techniques could expose the missile to circumvention of the adaptive array jammer suppression capability.

## Random Search Algorithms

Some of the most easily implemented adaptive algorithms are based on a random search of optimum array performance.<sup>22,65,66,69,70</sup> Random search algo-

rithms are characterized by trial and error, where each successive trial is based on the previous trial's improvement or degradation of measured performance and a random perturbation. The simplicity of algorithm implementation results from the potential to directly measure performance (e.g., mean square error or output power) and to easily compute the next trial. In addition to simple implementation, this class of algorithms has the advantages of being applicable to a wide variety of antenna array geometries, element weighting methods, and performance measures. For example, when the performance surface is either highly complex or multimodal such that gradient-based algorithms are of questionable utility, the random search algorithms are particularly attractive. The price paid for such simple implementation and wide applicability is slow convergence. The random search algorithms are the slowest converging of the five classes of algorithms described herein. Convergence can be orders of magnitude slower than gradient-based algorithms.

The random search algorithms update the array weight vector with the general equation

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu_s(k) [\Delta \mathbf{w}(k)],$$

where  $\mu_s(k)$  establishes the step size between trials and  $\Delta \mathbf{w}(k)$  is a random weight change vector. There are several varieties of random search algorithms. In one of the more basic algorithms, linear random search (LRS),  $\mu_s(k)$  is a step size proportional to the change in measured performance such that

$$\mu_s(k) = \mu_0 \{ \beta[\mathbf{w}(k)] - \beta[\mathbf{w}(k) + \Delta \mathbf{w}(k)] \},$$

where  $\mu_0$  is a step size constant and  $\beta(\cdot)$  is measured performance for a given weight vector. For the LRS algorithms,  $\Delta \mathbf{w}(k)$  is a normally distributed random vector with zero mean and variance  $\sigma^2$ .

The accelerated random search (ARS) algorithm is an attempt to improve convergence by increasing the step size in directions of successful performance improvement.  $\mu_s(0)$  and  $\mathbf{w}(0)$  are initialized to appropriate values and  $\Delta \mathbf{w}(k)$  is chosen randomly with components

$$\Delta w_i(k) = \cos \theta_i + j \sin \theta_i,$$

where  $\theta_i$  is uniformly distributed between 0 and  $2\pi$ .

<sup>69</sup> R. L. Barron, "Guided Accelerated Random Search as Applied to Adaptive Array AMTI Radar," *Proc. Adapt. Ant. Syst. Workshop*, 11-13 Mar 1974, NRL Report 7803, I, pp. 101-112 (Sep 1974).

<sup>70</sup> A. E. Zeger and L. R. Burgess, "Adaptive Array AMTI Radar," *Proc. Adaptive Ant. Sys. Workshop*, 11-13 Mar 1974, NRL Report 7803, I, pp. 81-100 (Sep 1974).

If, after measuring the performance with weight  $\mathbf{w}(1)$  and  $\mathbf{w}(0)$ , there is an improvement, then  $\Delta\mathbf{w}(1)$  remains unchanged from  $\Delta\mathbf{w}(0)$  and  $\mu_0(1)$  is doubled from  $\mu_0(0)$ . This process continues until the performance between weight  $\mathbf{w}(k+1)$  and  $\mathbf{w}(k)$  degrades, at which time  $\mu_0(k)$  is set to the initialized value.  $\Delta\mathbf{w}(k)$  is chosen randomly and a new  $\mathbf{w}(k+1)$  is established. The guided accelerated random search (GARS) algorithm is similar to the ARS algorithm in that as long as performance is improving in a given weight direction, the step size is doubled. The primary difference between the two algorithms is that when performance degrades the GARS algorithm chooses a normally distributed perturbation weight vector  $\Delta\mathbf{w}$ , that changes in variance  $\sigma^2$ , as a function of performance.

The convergence time for random search algorithms is dependent on the signal environment, the number of adapted weights, and the tolerable steady state error relative to optimum performance. The signal environment, as characterized by the jammer power levels and locations, determines the make-up of the signal correlation matrix  $\mathbf{R}_{xx}$ . The eigenvalues of  $\mathbf{R}_{xx}$  set the limits on random search algorithm parameters to ensure stable convergence. In particular, the stability condition for the LRS algorithm is

$$1/\lambda_{\max} > \mu_0 \sigma^2 > 0,$$

where  $\lambda_{\max}$  is the maximum eigenvalue of correlation matrix  $\mathbf{R}_{xx}$ . The transient response of the adaptive weights in normal coordinates can be represented as a sum of exponentials with time constants

$$\tau_p = 1/2\mu_0 \sigma^2 \lambda_p, \quad p = 1, 2, \dots, N,$$

where  $\lambda_p$  is the  $p$ th eigenvalue of  $\mathbf{R}_{xx}$ . Accordingly, the mean square error (MSE) is a quadratic function of the adaptive weights and is exponential in nature. The MSE as a function of the number of adaptive iterations is referred to as a "learning curve" and has associated  $p$ th mode time constraints

$$\tau_{p_{mse}} = 1/4\mu_0 \sigma^2 \lambda_p, \quad p = 1, 2, \dots, N.$$

The  $p$ th mode learning curve time constant in independent data samples is

$$T_{p_{mse}} = k_s \tau_{p_{mse}},$$

where  $k_s$  is the number of samples required for each iteration of the adaptive process.

An adaptive algorithm does not converge until the slowest mode converges. Large eigenvalues yield faster convergence than smaller eigenvalues. However, it is the largest eigenvalue of  $\mathbf{R}_{xx}$  that establishes the maximum rate of adaptation as limited by stable convergence and steady state noise. Thus, the ratio of the maximum eigenvalue to minimum eigenvalue of  $\mathbf{R}_{xx}$  (i.e., eigenvalue spread) is a primary determinant of convergence speed.

The LRS algorithm parameters are further specified by the degree of excess MSE relative to the minimum MSE that can be tolerated when the adaptive process reaches steady state. The misadjustment caused by weight vector noise is defined as

$$M \triangleq (\xi_{actual} - \xi_{min}) / \xi_{min},$$

where  $\xi_{min}$  is the minimum achievable MSE and  $\xi_{actual}$  is the actual steady state MSE. The total misadjustment,  $M_{tot}$ , for the LRS algorithm consists of the random weight vector process and the deterministic misadjustment caused by weight perturbation. The total misadjustment is minimized for a given learning curve time constant such that

$$(M_{tot})_{min} = N \left| \left( \frac{1}{T_{p_{mse}}}_{av} \right) \right|^2,$$

where  $(T_{p_{mse}})_{av}$  is the average learning curve time constant. Accordingly, the average learning curve time constant in data samples can be represented as a function of optimized misadjustment

$$(T_{p_{mse}})_{av} = \left| \frac{N}{(M_{tot})_{min}} \right|^2$$

As indicated by the above relationship, the LRS algorithm parameters are a tradeoff between total misadjustment and average response speed, up to the point where the algorithm is unstable.

The convergence time for the LRS algorithm can vary considerably from the average learning curve time constant since convergence time is sensitive to eigenvalue spread of  $\mathbf{R}_{xx}$ . Bounds on convergence time can be established by noting that

$$N\lambda_{av} = tr[\mathbf{R}_{xx}],$$



where  $\text{tr}[\mathbf{R}_{xx}]$  is the trace of the autocorrelation matrix  $\mathbf{R}_{xx}$  and is equal to the total array input power. Multiplying the average learning curve time constant by the ratio of average to minimum eigenvalues establishes the LRS algorithm time constant in terms of the correlation matrix trace.

$$T_{LRS} = \frac{\text{tr}[\mathbf{R}_{xx}]}{N\lambda_{\min}} (T_{P_{\text{mse}}})_{\text{av}} = \frac{\text{tr}[\mathbf{R}_{xx}]}{\lambda_{\min}} \frac{N}{[(M_{\text{tot}})_{\min}]^2}.$$

When the ratio of  $\text{tr}[\mathbf{R}_{xx}]$  to  $\lambda_{\min}$  is large (i.e., large eigenvalue spread) the convergence times are long.

As an example, consider an eight-element adaptive array. A signal correlation matrix trace of 1000 could easily be encountered. When designed to yield a minimum total misadjustment of 10%, the average learning curve time constant is 6400 samples. However, the convergence time constant will exceed  $8 \times 10^5$  samples when the eigenvalue spread reaches a ratio of 1000. This example illustrates the extremely long convergence times associated with the LRS algorithm even with a small number of adaptive weights. The ARS and GARS algorithms can yield improved times to convergence (maybe an order of magnitude) but still exhibit the same sensitivities to eigenvalue spread and number of adaptive weights.

Since the random search algorithms require little computation, implementation on a missile seeker would not be difficult, provided an appropriate antenna feed system could be accommodated. The likely performance measure for a random search algorithm is signal power. The sensed signal environment would have to be filtered effectively to avoid nulling on desired signals. In addition, constraints would have to be implemented on the adaptive weight controls to insure that maximum gain is in the desired signal direction.

The convergence time for the random search algorithms does not appear to be adequate for the defined adaptive missile seeker antenna. When a eigenvalue spread of  $1 \times 10^4$  in  $\mathbf{R}_{xx}$  is assumed for a minimal 8-channel adaptive system, the convergence times exceed  $8 \times 10^5$  samples. This even includes a factor of 10 improvement resulting from using an accelerated search algorithm. This response time is far in excess of the  $1 \times 10^4$  samples to converge as required.

## Gradient-Based Algorithms

Some of the more popular adaptive algorithms are based on a gradient approach to optimization. These algorithms seek either the minimum or maximum of a performance surface by adjusting the adaptive weights in directions corresponding to the perfor-

mance gradient. The two gradient-based adaptive algorithms addressed here are referred to as least mean squares (LMS) and maximum signal-to-interference-plus-noise ratio (MSN). Both algorithms have been the subject of much research and experimentation<sup>1,2</sup> and they comprise a set of proven techniques for adaptive arrays. They offer a good compromise between performance and complexity for many systems.

The LMS and MSN algorithms are implemented as closed-loop correlation processors. That is, each weight change is based on the previous weight value and the correlation between input and output signals. The LMS algorithm developed by Widrow<sup>3</sup> seeks to minimize the mean square error between the desired array output and the actual array output. It correlates an error signal with the array element signals when computing the next set of element weights. The discrete time equation for the LMS algorithm is

$$\mathbf{w}(i+1) = \mathbf{w}(i) + 2\Delta_e \mathbf{e}(i) \mathbf{x}^*(i),$$

where

$$\mathbf{e}(i) = d(i) - y(i),$$

$$y(i) = \mathbf{w}^T(i) \mathbf{x}(i) = \sum w_k(i) x_k(i),$$

$\mathbf{w}(i)$  is the adaptive weight vector, and  $\Delta_e$  is a step size constant. The error signal,  $\mathbf{e}(i)$ , results from differencing the array output,  $y(i)$ , from a desired reference signal,  $d(i)$ . The array output is the summation of the element signals,  $\mathbf{x}(i)$ , weighted with weights,  $\mathbf{w}(i)$ . The reference signal required by the LMS algorithm should be correlated with the desired received signal. Although this is often possible with communication systems, there is usually not enough a priori information about radar returns to provide an adequate reference signal.

The MSN algorithm developed by Howells and Applebaum<sup>1,4</sup> does not require a reference signal and is generally more suitable for radar applications. The MSN algorithm attempts to maximize S/IN and uses the correlation between the array output and element signals to compute weight values. Its digital implementation is described by the following equation:

$$\begin{aligned} \mathbf{w}(i+1) = & \mathbf{w}(i) [1 - (1/\tau_0)] - (\gamma/\tau_0) \mathbf{x}^*(i) y(i) \\ & + (b^*/\tau_0), \end{aligned}$$

where

$$\gamma = k_m^2 G$$

$$y(i) = \mathbf{w}^T(i) \mathbf{x}(i).$$

The analog form of the MSN algorithm is illustrated in Fig. 8. It is implemented with a smoothing filter that has an equivalent time constant,  $\tau_0$  that is expressed in number of data samples. The constant  $\gamma$  is the product of mixer conversion constant,  $k_m$ , and loop gain,  $G$ . Unlike the LMS algorithm, the MSN algorithm includes an array beam steering vector,  $\mathbf{b}^*$ . The desired look direction of the adaptive array must be known to compute the steering vector.

The LMS and MSN algorithms, though developed from different backgrounds, are actually quite similar. Each algorithm converges to the optimum Weiner solution with weight vectors that differ only by a multiplicative constant. The convergence properties of the two algorithms, including transient response and algorithm misadjustment, are nearly identical. The primary difference between the two algorithms is the method of array steering. The LMS algorithm does not require knowledge of the desired signal direction, but is steered appropriately with a refer-

ence signal. The MSN algorithm is steered by a vector that requires knowledge of the desired array look direction. Since the MSN algorithm does not require a reference signal, it is more easily applied to radar systems and will be discussed further for application to a missile seeker.

The convergence properties of the MSN algorithm is dependent on the signal environment. However, the environment sensitivity is considerably less than that of the random search algorithms. The adaptive weights, when expressed in orthonormal coordinates, have a transient response to a step change in the environment that consists of a sum of exponentials with each exponential having a time constant proportional to an eigenvalue of  $\mathbf{R}_{xx}$ . The time constant related to the  $p$ th eigenvalue of  $\mathbf{R}_{xx}$ ,  $\tau_p$ , takes the form

$$\tau_p = \tau_0 / (1 + \gamma \lambda_p),$$

where  $\tau_0$  and  $\gamma$  are specified MSN algorithm parameters and  $\lambda_p$  is an eigenvalue. It is evident that for a given set of algorithm parameters, the effective time constant for the adaptive weights is limited by the minimum eigenvalue of  $\mathbf{R}_{xx}$ , or

$$\tau_{eff} = \tau_0 / (1 + \gamma \lambda_{min}) \approx \tau_0 / \gamma \lambda_{min}.$$

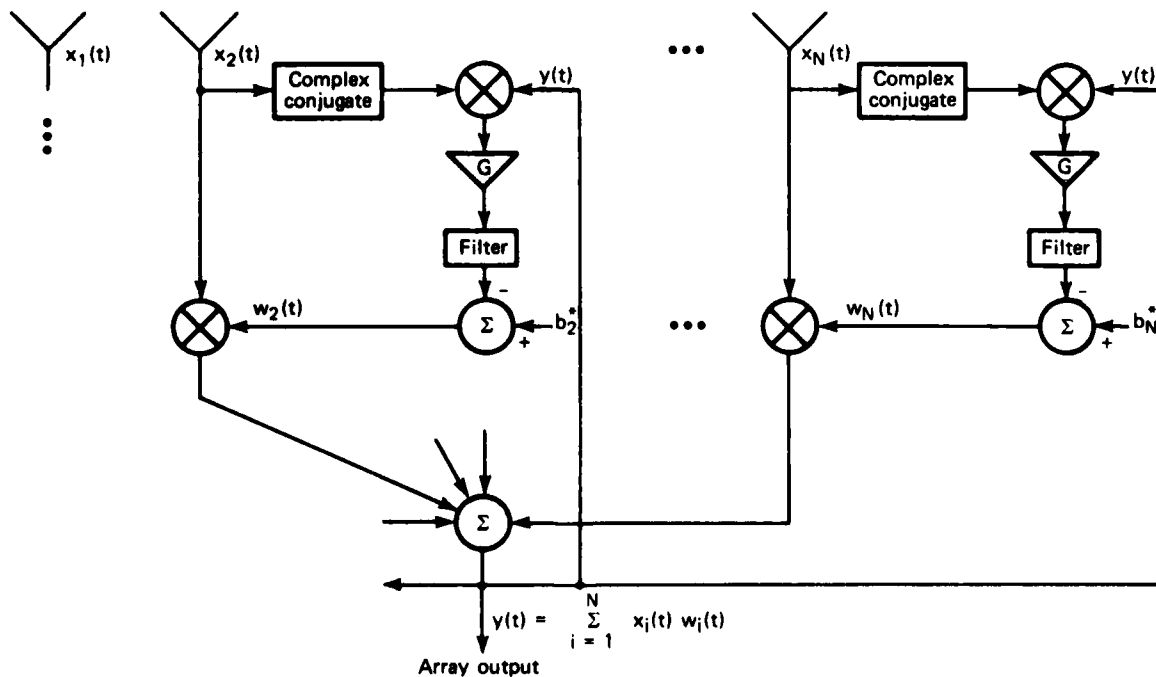


Figure 8 — MSN adaptive processor in analog form.

Since the output noise power is a quadratic function of the weights, the time constant for the S/IN performance of the MSN algorithm is half that of  $\tau_{eff}$ :

$$\tau_{MSN} = \tau_{eff}/2$$

The smoothing filter time constant,  $\tau_0$ , and loop gain,  $\gamma$ , are selected in accordance with permissible misadjustment. The misadjustment for algorithms that maximize S/IN is defined as excess noise power relative to minimum achievable noise power and is analogous to misadjustment when defined in terms of mean square error. The MSN algorithm misadjustment can be expressed approximately as<sup>22</sup>

$$M_{SNR} \approx (\gamma/2\tau_0) tr(\mathbf{R}_{nn}),$$

where  $tr[\mathbf{R}_{nn}]$  is the trace of the noise correlation matrix or equivalently the total noise power impinging on the array,  $P_N$ , from all sources, including jammers and thermal noise, and is computed from

$$tr[\mathbf{R}_{nn}] = [\mathbf{n}^T(i)\mathbf{n}^*(i)] = \sum_{k=1}^N E[|n_k|^2] \triangleq P_N.$$

MSN algorithm parameters chosen to establish a given level of misadjustment results in a time constant of

$$\tau_{MSN} = tr[\mathbf{R}_{nn}]/4M_{SNR}\lambda_{min}$$

When the eigenvalue spread is large (i.e.,  $tr[\mathbf{R}_{nn}]$  is large compared to  $\lambda_{min}$ ), a long time constant results. However, the convergence time for the MSN algorithms can easily be two or more orders of magnitude faster to converge than the LRS algorithm when the misadjustments are equal:

$$T_{LRS}/\tau_{MSN} = 4N/(M_{tot})_{min},$$

where  $N$  is the number of array elements and  $(M_{tot})_{min}$  is the total misadjustment for the LRS algorithm for a given set of algorithm parameters.

The MSN time constant for an 8-element array with a 20% misadjustment and ratio of total noise power to  $\lambda_{min}$  of  $1 \times 10^4$  is  $1.25 \times 10^4$  independent

time samples. This is slightly greater than the  $1 \times 10^4$  samples to converge that was established as the requirement for a missile seeker. The convergence time is improved by a factor of 160 over an LRS algorithm under similar conditions.

The MSN algorithm can be relatively simple to implement with only moderate complexity. One distinct advantage of this algorithm is the ability to implement it with digital hardware, analog devices, or a combination of the two. Since the algorithm is a closed-loop processor, it is relatively tolerant of hardware errors and requires low multiplication accuracy. When the MSN algorithm is implemented with digital hardware, a complex multiplication is required per sample for each adaptive channel or, equivalently,  $N$  complex multiplications per sample for an  $N$ -element array. An 8-element array with a sampling interval of 100 ns would require a digital processor capable of  $8 \times 10^7$  complex multiplications per second. However, an analog implementation of the same algorithm would require only eight analog loops, each with a 10 MHz bandwidth. Adaptive loops for a LMS algorithm (which is functionally equivalent to an MSN algorithm loop) have already been implemented in small thin-film hybrid modules.<sup>6</sup> One module corresponds to one adaptive loop. This kind of miniturization and modularization would simplify the implementation of a MSN algorithm and allow the use of a relatively large number of adaptive loops (more than 10).

Since the missile antenna is required to form a  $\Sigma$  beam, a  $\Delta_{az}$  beam, and a  $\Delta_{el}$  beam, provisions must be made for the MSN algorithm to optimally form each beam. It has been shown<sup>18</sup> that to optimally form each of these beams the MSN algorithm need only be modified with a different steering vector,  $\mathbf{b}^*$ , for each beam. Therefore, one set of adaptive control loops could be used to form three separate beams by modifying each beam with a different steering vector. The steering vector is computed to form appropriate sum and difference beams to maximize target detection and target angle estimation angle respectively.

As previously discussed, the MSN algorithm is inherently a closed-loop process. However, it is desirable to inhibit weight changes (i.e., pattern changes) during the missile's Doppler dwell time. Varying these weights during estimation of target Doppler shift not only can result in additional noise in the Doppler measurement but also can result in a jammer's deliberate exploitation of pattern changes to degrade the Doppler measurement. The MSN algorithm could be implemented to allow the holding of weight values for specified periods of time. This needs to be investigated further, but could result in increased convergence times. Another method would

be to introduce a slave weight control unit. The MSN algorithm would be implemented as described previously and allowed to adapt continuously. The continually updated weights would be used to form the array output for use by the control-loop processor only. The array output used by the missile receiver would be formed with a separate set of weights that are updated periodically with the values of the continually updated weights. Although this method would reduce somewhat the hardware error tolerance of the MSN algorithm, it would allow the quickest possible adaption.

The MSN algorithm's speed of convergence is on the borderline of acceptability for a high performance missile implementation. Although modifications to the MSN algorithm (such as hard limiting the array element inputs to the control loop) can improve transient response in some situations, the worst case situations involving large eigenvalue spreads in the noise correlation matrix,  $\mathbf{R}_{nn}$ , can still lead to long convergence times. However, the implementation simplicity and flexibility of the MSN algorithm makes it attractive from a hardware perspective for missile implementation. In addition, there is a large body of knowledge and experimentation that is directly applicable to the MSN algorithm. Therefore, the MSN algorithm should not be dismissed out of hand as a candidate algorithm for an adaptive missile seeker antenna. One should approach such an implementation cautiously with the knowledge that the area of highest risk is probably convergence speed.

### Sample Matrix Inversion Approach

By directly computing adaptive weight values from a sample covariance matrix of the signal environment, rapid adaptive array convergence can be achieved. Although this method requires the inversion of the sample covariance matrix, convergence rates can be several orders of magnitude faster than correlation loop algorithms. In addition, the convergence properties of the Sample Matrix Inversion approach (SMI) are theoretically independent of the signal environment (i.e., eigenvalue spread of the sample covariance matrix).<sup>22,71</sup>

The primary disadvantage of the SMI approach is the complexity required of the adaptive processor. The formation of the sample covariance matrix and subsequent matrix inversion are arithmetic-operation-intensive and are accomplished practically only with digital hardware. Thus, an SMI-based adaptive

processor can be considerably more complex than an MSN adaptive processor. Because the SMI approach is a direct weight computation, the adaptive processor is open-loop and is less tolerant of hardware errors than the MSN algorithm. The use of a SMI adaptive processor is typically suggested when rapid adaptive array convergence is a critical requirement, as in airborne radars.<sup>12</sup>

The SMI computation is derived directly from the equation for optimum array weighting. The weight vector that optimizes S/IN performance of an adaptive array is given by

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xd}$$

where  $\mathbf{r}_{xd}$  is the cross-correlation vector between the signal vector,  $\mathbf{x}(t)$  and the desired reference signal,  $\mathbf{d}(t)$ , and  $\mathbf{R}_{xx}$  is the covariance matrix of  $\mathbf{x}(t)$ . Since  $\mathbf{R}_{xx} = \mathbf{R}_{nn}$  when the desired signal is absent, the SMI weights are computed from

$$\mathbf{w}_{SMI} = \hat{\mathbf{R}}_{nn}^{-1} \mathbf{r}_{xd} = \hat{\mathbf{R}}_{nn}^{-1} \mathbf{b}^*$$

where  $\hat{\mathbf{R}}_{nn}$  is the sample covariance estimate of the interference and noise environment and  $\mathbf{b}^*$  is a beam-steering vector specified by target angle of incidence.<sup>71</sup> The sample covariance matrix,  $\mathbf{R}_{nn}$ , is formed from samples of the signal vector,  $\mathbf{x}(t)$ , when the desired signal is absent and is given by

$$\hat{\mathbf{R}}_{nn} = \frac{1}{M} \sum_{i=1}^M \mathbf{x}^*(i) \mathbf{x}'(i)$$

where  $\mathbf{x}(i)$  is the  $i$ th signal vector time sample out of  $M$  samples.

The convergence properties of the SMI approach can be evaluated on the basis of S/IN performance relative to optimum with the random variable

$$\rho = (s/n) / SN_0$$

where  $s/n$  is the S/IN for a given weight vector computed by the SMI method and  $SN_0$  is the optimum S/IN (i.e.,  $SN_0 = \mathbf{s}' \mathbf{R}_{nn}^{-1} \mathbf{s}^*$ ). In Ref. 71 the probability distribution of  $\rho$  is shown to be a incomplete beta distribution with mean

<sup>71</sup> J. S. Reed, J. D. Mallett, and L. E. Brennan, "Rapid Convergence Rate in Adaptive Arrays," *IEEE Trans. Aerosp. Electron. Syst.*, AES-10, pp. 853-863 (Nov 1974).

$$E[\rho] = \rho = \frac{M + 2 - N}{M + 1}$$

and variance

$$\text{var}(\rho) = \frac{(M + 2 - N)(N - 1)}{(M + 1)^2(M + 2)},$$

where  $N$  is the number of adaptive weights. The mean of  $\rho$  indicates that SMI-computed adaptive weights converge to within 3 dB of the optimum S/IN for a sample number  $M > 2N$  (assuming a value of  $N$  much larger than 2). Thus the convergence time for the SMI method is established as

$$\tau_{SMI} = 2N.$$

Of significance is the observation that the normalized S/IN,  $\rho$ , has a mean and variance independent of the sample covariance matrix eigenvalues. Thus, SMI convergence is insensitive to the interference environment, unlike gradient-based or random-search algorithms.

The complexity of an SMI-based adaptive processor can be established by counting the number of complex multiplications required to compute the adaptive weight vector. Since the sample covariance matrix is Hermitian (i.e.,  $\hat{\mathbf{R}}_{nn}^T = \hat{\mathbf{R}}_{nn}^*$ ), the computational requirements for sample matrix formation and inversion are somewhat eased. Even so, the sample covariance matrix formation requires  $MN(N + 1)/2$  complex multiplications and the matrix inversion requires  $N^3/2 + N^2$  complex multiplications. The weight vector computation requires an additional  $N^2$  complex multiplications. Letting  $M = 2N$  for a mean convergence within 3 dB of optimum, the total number of complex multiplications required to compute the adaptive weight vector is  $(3/2)N^3 + 3N^2$ . Although other methods that compute weight values directly from the sample covariance matrix and beam steering vector can result in somewhat reduced computational requirements,<sup>72</sup> the required number of complex multiplications is still on the order of  $N^3$ . The complex multiplications require relatively high accuracy; otherwise, S/IN performance will be sensitive to eigenvalue spread of the sample covariance matrix.<sup>22</sup> An ill-conditioned sample covariance matrix (i.e., large eigenvalue spread) can result in

inaccurate matrix inversion and S/IN degradation, unless the arithmetic computations are carried out with sufficient accuracy.

An eight-element adaptive array with SMI processing converges (within 3 dB of optimum) in 16 independent time samples and requires approximately 1000 complex multiplies to compute the resulting weight values. Compared to the  $1.25 \times 10^4$  sample MSN time constant for a similar array with an eigenvalue spread of  $1 \times 10^4$ , the SMI convergence rate is a dramatic improvement. Unlike the MSN algorithm, the continual recomputation of the adaptive weight vector with every independent sample is not required. The processing requirements for the SMI algorithm would be excessive otherwise. Rather, the SMI computation is performed on blocks of independent samples that are a fraction of those available. In this way, the adaptive processor selects independent samples and updates the weight vector at appropriate intervals. As long as the number of independent samples in each computation is sufficient for a specified degree of convergence, and the interference environment does not change significantly between weight updates, the adaptive array will continue to maximize S/IN. The processing requirement will be reduced according to the fraction of samples processed between weight updates. Thus, the processing requirement for SMI-based eight-element adaptive array when updating adaptive weights every  $1 \times 10^4$  samples and using 16 independent time samples is  $0.1/\tau_c$ , where  $\tau_c$  is the minimum interval between independent samples. For a  $\tau_c$  of 100 ns, the processor is required to perform approximately one million complex multiplications per second. An equivalent MSN adaptive array (mentioned in a previous example) implemented in digital hardware requires approximately 80 million complex multiplications per second. For a digital implementation of a modest adaptive array, the SMI method requires less processor capability. However, as the number of adaptive weights increases, the SMI processing requirements will exceed those of the MSN algorithm. SMI processing requirements are on the order of  $N^3$  and MSN processing requirements are on the order of  $N$ .

To form the sum and difference beams as required for the missile seeker, only a modest increase in processor complexity is required, since the weight computation for each beam differs only in the beam-steering vector.<sup>18</sup> The inversion of the sample covariance matrix need not be repeated for each beam. The difference beam-steering vectors are chosen to optimize angle estimation accuracy. With the formation of three beams and sum beam S/IN convergence within 1 dB of optimum,  $5N$  independent samples are required and three weight computations. Thus, each

<sup>72</sup>J. S. Bailey, "Gram Schmidt Decomposition," *Proc. 1980 Adaptive Ant. Symp.*, Rome Air Development Center, RADC-TR-80-378 (Dec 1980).

set of weight updates requires  $(11/2)N^1 + (13/2)N^2$  complex multiplications. For a maximum processing capability of 200 million complex operations per second and weight updating every ten thousand samples, the maximum number of adaptive elements that can be processed with the SMI method is found, approximately, by

$$N \approx 7000 \tau_e^{-1/3}.$$

When the minimum interval between independent samples is 100 ns, 32 adaptive elements can be accommodated to form three separate beams. This is considerably less than the approximately 200 separate elements that is potentially available with a Ku-band planar array of missile dimensions. Therefore, if an adaptive array with SMI processing is to be implemented in a missile seeker, the antenna array must be reduced in dimension. Previous discussion suggested ways to reduce array dimension while maximizing adaptive pattern flexibility and performance.

Because an SMI adaptive missile antenna is by nature open-loop, errors in hardware will be less tolerable than with a closed-loop processor. In addition to minimizing the hardware errors with appropriate design and component selection, an automatic calibration may be required periodically. An open-loop adaptive processor is suited to missile signal processing. By holding the adaptive weights constant during the Doppler dwell time, the antenna pattern will remain constant so as not to contribute to Doppler processing errors. During the weight-holding period, independent samples can still be taken and weight computation can proceed unimpeded. Like other adaptive algorithms that maximize S/IN without using a reference signal, independent samples taken when the desired signal is absent (in time or frequency) is desirable. Otherwise, degraded performance and slowed convergence may result.

### Recursive Methods

The computational requirements of SMI processing can be simplified by recursively performing matrix inversion instead of directly calculating matrix inverses. The desirable SMI properties of rapid convergence and insensitivity to signal covariance matrix eigenvalue spread can still be realized with recursive methods. Derived from least squares estimation techniques, recursive methods of adaptive antenna processing are similar in many respects to Kalman filtering.<sup>22,72,73</sup> Recursive processors are intended primarily

ly for digital implementation, and can be developed for a variety of signal weighting schemes.

Recursive methods involve periodic computation and updating of array weights. The update equation for a weighted least squares error processor<sup>22</sup> is

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mathbf{P}(k) [\mathbf{b}^* - \mathbf{x}^T(k+1)\mathbf{w}(k)\mathbf{x}^*(k+1)]}{[\alpha + \mathbf{x}^T(k+1)\mathbf{P}(k)\mathbf{x}^*(k+1)]},$$

where

$$\mathbf{P}(k+1) = \frac{1}{\alpha} \left\{ \mathbf{P}(k) - \frac{\mathbf{P}(k)\mathbf{x}^*(k+1)\mathbf{x}^T(k+1)\mathbf{P}(k)}{[\alpha + \mathbf{x}^T(k+1)\mathbf{P}(k)\mathbf{x}^*(k+1)]} \right\}.$$

The constant  $\alpha$  ( $0 \leq \alpha \leq 1$ ) serves to de-emphasize previous data as new data are used in the computation. The above equation is applicable when the desired signal direction is known. The algorithm is initiated by assuming an initial value for the weight vector  $\mathbf{w}(0)$  and initial  $\mathbf{P}(0)$ .

Another formulation of a recursive processor is to periodically update the sample covariance matrix inverse  $\mathbf{R}_{xx}^{-1}$ . This technique is described by

$$\mathbf{w}(k+1) = \hat{\mathbf{R}}_{xx}^{-1}(k)\mathbf{b}^*,$$

where

$$\hat{\mathbf{R}}_{xx}^{-1}(k+1) = \frac{1}{\alpha} \left\{ \hat{\mathbf{R}}_{xx}^{-1}(k) - \frac{\mathbf{R}_{xx}^{-1}(k)\mathbf{x}^*(k+1)\mathbf{x}^T(k+1)\hat{\mathbf{R}}_{xx}^{-1}(k)}{\alpha + \mathbf{x}^T(k+1)\mathbf{R}_{xx}^{-1}(k)\mathbf{x}^*(k+1)} \right\}$$

Each update computation requires  $(7/4)N^2 + (9/4)N$  complex multiplies. When updated continually, an SMI processor requires  $(1/2)N^1 + (3/2)N^2$  complex multiplies per update. Thus recursive processing is advantageous from a computational viewpoint when continual updating is required. In addition, recursive methods can be more tolerant of hardware inaccuracies. When processing data in blocks, the SMI

<sup>73</sup>C. A. Baird, "Recursive Processing for Adaptive Arrays," *Proc. Adaptive Ant. Syst. Workshop*, 11-13 Mar 1974, I, NRI Report 7803, pp. 163-182 (Sep 1974).

method may actually require less computation. Recursive methods may still be advantageous, however, since computational load can be more evenly distributed in time.

Because of its digital processing and algorithm flexibility, a recursive processor could be readily implemented in a missile seeker. The implementation of a recursive processor would be very similar to that of a SMI processor. The primary difference between the two techniques is that method of calculating the sample covariance matrix inverse. Since the recursive algorithm requires more computations for a fixed number of samples than a block-processed SMI algorithm, a recursive processor can accommodate fewer adaptive channels than an SMI processor when updates are required relatively infrequently. Alternately, when an update is required for each signal vector sample, the recursive algorithm is capable of supporting more adaptive channels than an SMI processor for a given processor throughput capability.

### Cascade Preprocessors

Adaptive array convergence can be improved for several adaptive processors by implementing a preprocessing network. White<sup>74,75</sup> initially advocated a cascade preprocessor network to improve the convergence rate of gradient type algorithms. A cascade preprocessor transforms the array element signals into signal components that have a correlation matrix with reduced eigenvalue spread. Thus, a cascade preprocessor can accelerate convergence for an eigenvalue sensitive adaptive processor. Although a cascade preprocessor requires increased hardware and processing, it has proven to be a flexible approach to rapid adaptive array convergence.<sup>22</sup>

The Nolen network preprocessor and Gram Schmidt orthogonalization preprocessor are the better known cascade preprocessor types. A Nolen cascade network transforms the array signal vector into orthogonal eigenvector beam components. If, for example, a Nolen network preprocessor is followed by a gradient-based adaptive processor, only one iteration is required of the gradient algorithm to achieve the optimum response regardless of the array signal correlation matrix eigenvalue spread. However, the Nolen network requires a considerable number of iterations before the network is adjusted to the proper transformation<sup>22</sup> and is typically not used in practice. The Gram Schmidt orthogonalization pre-

processor requires a relatively small number of iterations to achieve the desired transformation and can be implemented practically. It resolves the array element signals into orthogonal components with reduced eigenvalue spread, which simplifies the subsequent adaptive processing. Because of its practicality and widespread acceptance,<sup>10</sup> the following discussion on cascade preprocessors will be limited to the Gram Schmidt orthogonalization preprocessor.

The optimal weight vector solution derived previously is given by

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xd}$$

where  $\mathbf{r}_{xd}$  is the cross-correlation between the array signal vector,  $\mathbf{x}(t)$ , and the desired reference signal,  $\mathbf{d}(t)$ , and  $\mathbf{R}_{xx}$  is the array signal correlation matrix,  $E[\mathbf{x}(t)\mathbf{x}^*(t)]$ . For an appropriate linear transformation matrix  $\mathbf{Q}$ ,  $\mathbf{R}_{xx}$  can be transformed into a diagonal correlation matrix:

$$\mathbf{R}_{xy} = \mathbf{Q}\mathbf{R}_{xx}\mathbf{Q}^*$$

The optimal weight vector solution can then be expressed as

$$\mathbf{w}_{opt} = \mathbf{Q}^*\mathbf{R}_{xy}^{-1}\mathbf{Q}\mathbf{r}_{xd}$$

The transformation matrix is found by the well known Gram Schmidt orthogonalization process.<sup>76</sup> After transformation, the array signals are orthogonal to each other and have a correlation matrix  $\mathbf{R}_{xy}$  with non-zero elements only on the diagonal. The subsequent processing of the transformed signals to compute the optimum weight vector solution (whether by matrix inversion, gradient, or other technique) is greatly simplified.

Gram Schmidt orthogonalization can be achieved with a multilevel network, as illustrated in Fig. 9. Each node of the network achieves orthogonality between the transformed output signal and a reference signal. The reference signal for a given network level is a transformed signal from the previous level. Thus the transformation at each node (signals indexed as in Fig. 9) can be expressed as

$$v_n^{k+1} = v_n^k - u_{k(n-1)} v_k^k, \quad \text{for } (k+1) \leq n \leq N,$$

<sup>74</sup>W. D. White, "Accelerated Convergence Techniques" *Proc. Adaptive Ant. Syst. Workshops*, 11-13 Mar 1974, I, NRL Report 7803, pp. 171-215 (Sep 1974).

<sup>75</sup>W. D. White, "Cascade Preprocessors for Adaptive Antennas," *IEEE Trans. Ant. Propag.*, **AP-24**, pp. 670-684 (Sep 1976).

<sup>76</sup>H. Anton, *Elementary Linear Algebra*, John Wiley & Sons (1977).

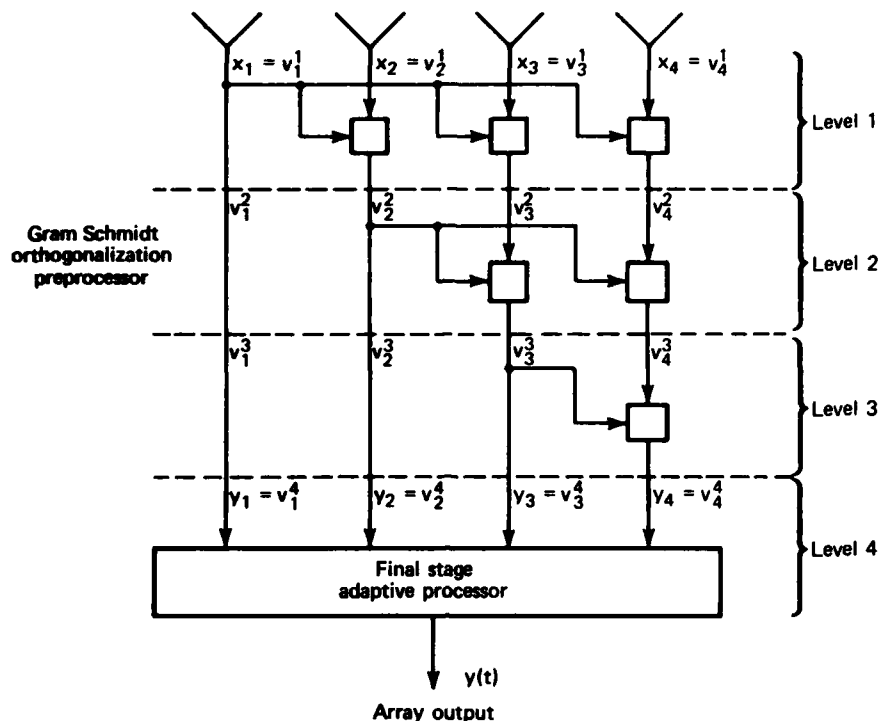


Figure 9 — Gram Schmidt orthogonalization network with final adaptive processing stage.

where  $N$  = number of array elements and

$$u_{k(n-1)} = E(v_k^{k*} v_n^{k+1}) / E(v_k^{k*} v_k^k)$$

in steady state. Gram Schmidt orthogonalization can be achieved with  $N(N-1)/2$  separate transformations (nodes) for an  $N$ -element array.

Each node of a Gram Schmidt orthogonalization preprocessor can be realized with a Howells-Applebaum MSN adaptive loop, since the loop produces an output that in steady state is uncorrelated with a given reference signal. Such a network could then be followed by  $N$  additional MSN adaptive loops independently applied to each of the transformed signals to adaptively form the final array output. The complete adaptive processor, including Gram Schmidt preprocessor, requires  $N(N+1)/2$  adaptive loops. A corresponding digital implementation requires  $N(N+1)$  complex multiplies per iteration.

When the adaptive loops in a Gram Schmidt preprocessor and MSN processor operate without noise, convergence can be achieved in  $N$  independent samples.<sup>22</sup> However, in practice several input signal samples are averaged for each iteration and the loop gains are set so as to reduce the effects of noise. Previous simulations<sup>22</sup> indicate that adaptive array con-

vergence within 3 dB of optimum can be achieved in about  $30N$  samples with acceptable loop noise. Convergence does, however, vary with the implementation and the optimum S/I/N. An 8-element adaptive array implemented with a Gram Schmidt network would require 36 separate adaptive loops and would converge in approximately 240 samples relatively independent of the interference environment.

Gram Schmidt orthogonalization makes a flexible adaptive processor implementation with relatively rapid adaption.<sup>72,77</sup> The Gram Schmidt network can be implemented with analog or digital adaptive loops. It can also be implemented as an open-loop digital process. When implemented as a closed-loop process, the Gram Schmidt network is relatively tolerant of hardware errors. An open-loop implementation, however, allows for more flexibility with multiple beams and periodic weight updating.

Adaptively forming the sum and difference beams of a missile seeker with the Gram Schmidt orthogonalization process would require one preprocessor network to transform the signals. A separate adap-

<sup>77</sup> W. C. Liles, R. R. Ritchey, and J. W. Demmel, "Design Trade-offs and Implementation of Gram Schmidt Adaptive Arrays," *Proc. 1980 Adaptive Ant. Symp.*, Rome Air Development Center, RADC-TR-80-378, I, pp. 220-234 (Dec 1980).



tive processor for the final stage would use three separate steering vectors to form the three beams. For a 200 element fully adaptive array, 20,500 loops would be required. Although convergence could be achieved in 6000 samples, the number of loops is excessive, especially if implemented with analog loops. A digital implementation is limited by processor capability. To form three beams,  $N^2 + 5N$  operations per process iteration is required. For convergence in  $10N$  iterations,  $10N^3 + 50N^2$  complex multiplies are required. The direct sample matrix inversion (SMI) can converge to within 1 dB in  $(11/2)N^3 + (13/2)N^2$  complex multiplies. When the data can be processed in blocks, as may be the case for a missile seeker, the SMI technique requires a smaller number of computations to converge. A Gram Schmidt preprocessor is advantageous when continual updating is required. The data processing required of a Gram Schmidt network can also be distributed more evenly in time than the SMI approach.

Gram Schmidt orthogonalization appears to be a viable approach to adaptively forming the beams of a missile seeker antenna. A digital open-loop implementation is probably the better suited method of implementing a Gram Schmidt network for the missile application. Like an SMI processor, processing capability will limit the number of antenna channels that can be adapted.

### Adaptive Algorithm Suitability

The previously described adaptive algorithms are briefly summarized in Table 3. In general, it appears

that a digitally implemented algorithm is more suited for a missile seeker. A digital adaptive processor is more flexible than an analog processor and will readily conform to the unique weight updating needs and multiple beam formation requirements of an adaptive missile seeker antenna. The one exception could be an analog implementation of the maximum signal-to-interference-plus-noise (MSN) algorithm. Although the MSN algorithm's convergence is slower than desired, an analog implementation with integrated correlation loop modules could yield a small size and low cost adaptive processor. Such an implementation would sacrifice the flexibility and rapid convergence achieved with other digital processors.

Digital adaptive processors based on sample matrix inversion, cascade preprocessing, or recursive methods are suitable for the missile application (when implemented digitally, those algorithms are similar). They basically find the inverse of the sample covariance matrix using somewhat different methods. Each of the algorithms is computation intensive and will require a digital processor with state-of-the-art capability. When updating weights on every independent sampling period, a cascade preprocessor requires the least processing capability of the three algorithms. However, when weight updating is required only periodically, such that block processing of sampled data is appropriate, the direct sample matrix inverse approach can require the least processing capability. Recursive processing distributes the processing load more evenly than the other two algorithms and may actually be advantageous.

Table 3 — Adaptive algorithm summary

Adaptive algorithm type	Convergence speed <sup>1,2,3,4,5</sup> (number of independent samples)	Number of complex multiplies per input sample to update	Number of complex multiplies to converge	Salient features	Missile seeker implementation
Random search (linear random search) <sup>6,7</sup>	$\frac{tr[R_{NN}]}{\lambda_{min}} \frac{N}{M_{tot}^2}$  ( $3.2 \times 10^7$ ) <sup>10,11,12</sup>	—	—	Easy to implement. Can accommodate multimodel performance surfaces. Slow convergence depending on eigenvalues of $R_{NN}$ . Digital, analog, or hybrid.	Convergence excessively slow.
Gradient-based (MSN algorithm)	$\frac{tr[R_{NN}]}{4\lambda_{min}M_{tot}}$  ( $2.5 \times 10^4$ ) <sup>10,11,12</sup>	$N$  (32) <sup>10</sup>	$\frac{tr[R_{NN}]N}{4\lambda_{min}M_{tot}}$  ( $8 \times 10^5$ ) <sup>10,11,12</sup>	Established and proven technique. Relatively simple implementation. Tolerant of hardware errors. Convergence speed sensitive to eigenvalues of $R_{NN}$ . Digital, analog, or hybrid.	Borderline convergence speed. Analog implementation inflexible. Digital implementation yields no computational advantage.
Sample matrix inversion <sup>8</sup> (SMI)	$2N$  (64) <sup>10</sup>	$\frac{1}{2}N^3 + \frac{3}{2}N^2$  (17920) <sup>10</sup>	$\frac{3}{2}N^3 + 3N^2$  ( $5.2 \times 10^4$ ) <sup>10</sup>	Rapid convergence. Precise matrix inversion required. Efficient weight computation when input data is processed in blocks. Digital.	More than adequate convergence speed. Digital data block processing. State-of-the-art computational requirements.
Recursive (updated sample covariance matrix) <sup>9</sup>	$2N$  (64) <sup>10</sup>	$\frac{7}{4}N^2 + \frac{9}{4}N$  (1864) <sup>10</sup>	$\frac{7}{2}N^3 + \frac{9}{2}N^2$  ( $1.2 \times 10^5$ ) <sup>10</sup>	Similar to SMI. Rapid convergence. Some tolerance to hardware inaccuracies. Efficient when updating weights with each input sample. Digital.	More than adequate convergence speed. Update processing distributes computational load evenly. State-of-the-art computational requirements.
Cascade pre-processor <sup>8,9</sup> (Gram Schmidt orthogonalization)	$30N$  (960) <sup>10</sup>	$N^2 + 5N$  (1184) <sup>10</sup>	$10N^3 + 50N^2$  ( $3.8 \times 10^5$ ) <sup>10</sup>	Similar to SMI. Moderately rapid convergence. Some tolerance to hardware inaccuracies. Flexible implementation. Digital, analog, or hybrid.	Adequate convergence speed. Advantageous updating efficiency. State-of-the-art computational requirements.

Notes:

<sup>1</sup> $N$  = number of adaptive array elements

<sup>2</sup> $tr[R_{NN}]$  = trace of noise autocorrelation matrix (total noise + interference input power)

<sup>3</sup> $\lambda_{min}$  = minimum eigenvalue of  $R_{NN}$  corresponding to lowest power jammer

<sup>4</sup> $M_{tot}$  = steady state misadjustment

<sup>5</sup>Desired signal absent in input samples

<sup>6</sup>Ten times improvement possible with accelerated random search techniques

<sup>7</sup>Convergence speed equal learning curve time constant

<sup>8</sup>Convergence within 3 dB of optimum S/I/N

<sup>9</sup>Convergence speed established from simulation,<sup>31</sup> varies with implementation

<sup>10</sup> $N$  = 32 element array

<sup>11</sup> $tr[R_{NN}]/\lambda_{min} = 1 \times 10^4$

<sup>12</sup> $M_{tot} = 0.1$

## 8. CONCLUSIONS

Adaptive arrays are capable of improving dramatically the target detection and homing performance of a missile seeker in jamming environments. Realization of an adaptive missile seeker antenna is feasible in the next decade. However, the tradeoffs among cost, complexity, and performance remains to be seen. The primary limitations to adaptive array performance and realization in the foreseeable future are related to the state-of-the-art in technology. The fundamental adaptive array limitations are currently of less significance.

A fully adaptive Ku-band missile array with 185 separately controlled elements is not possible with the technology projected to be available in the early 1990's. Not only would the array feed and control exceed the physical limitations of the missile, but the digital processing requirement of about 10 billion complex multiplications per second significantly exceeds the projected capability of 200 million complex multiplications per second. Partially adaptive array techniques make possible missile implementation and can approach the performance of a fully adaptive array. By reducing the 185 elements of a Ku-band array to 28 subarrays formed from the column sums and row sums of elements, for example, the feed network can be greatly simplified and the processing requirement reduced to about 25 million complex multiplications per second. There are several subarray

methods and adaptive algorithms that can potentially satisfy the physical and processing constraints of a missile seeker.

Adaptive arrays can be sensitive to the limitations imposed by the signal environment and component errors. An adaptive array design that minimizes these effects will be essential. For example, channel tracking errors can limit jammer suppression to 20 dB or less when the RMS amplitude and phase errors exceed 0.6 dB and 4°, respectively. Closed-loop systems can compensate somewhat for component errors. Open-loop adaptive processors will probably require built-in and periodic calibration capability.

The particular errors that limit adaptive array performance vary with the implementation and fundamental characteristics of the system (e.g., frequency, bandwidth, and array aperture). The dominant errors expected in an adaptive missile seeker antenna must be identified and their values established. Thus experimentation with critical components and subsystems is essential. Digital simulation can support such experimentation by predicting the net effect of particular inaccuracies. Once established, realistic error models can be included in a simulation. A simulation can aid in the optimization of adaptive array performance relative to system configuration, cost, and expected errors.

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